

## Problem Set #5

Due Wednesday, March 18, 2026 @ 11:59 pm  
Submit as single pdf file to Canvas

Remember to review the **Guidelines for Problem Sets** on the course Webpage!

1. Recall the definition of a *McNugget number*: Chicken McNuggets originally came in boxes of 6, 9, or 20 McNuggets. A McNugget number is a positive integer that can be obtained by adding together orders of Chicken McNuggets, prior to consuming any.

(a) Prove that the following are McNugget numbers:

15, 35, 41, 42, 44, 45, 46, 47, 48, 49

(b) Prove that 43 is not a McNugget number

(c) Prove that every integer  $n \geq 44$  is a McNugget number

2. Define the Fibonacci sequence by  $f_0 = 0$ ,  $f_1 = 1$ , and  $f_n = f_{n-1} + f_{n-2} \quad \forall n \in \mathbb{N}, n \geq 2$

Let  $\varphi = \frac{1 + \sqrt{5}}{2}$  and  $\psi = \frac{1 - \sqrt{5}}{2}$ .

Prove that  $f_n = \frac{\varphi^n - \psi^n}{\sqrt{5}} \quad \forall n \in \mathbb{Z}, n \geq 0$

*Hint: It may be useful to observe that  $\varphi^2 = \frac{3 + \sqrt{5}}{2}$  and  $\psi^2 = \frac{3 - \sqrt{5}}{2}$ .*

3. If  $A$  and  $C$  are sets, prove that  $A = (A \cap C) \cup (A - C)$
4. Assume  $A$  and  $B$  are all subsets of some universal set  $U$ . Prove  $A \subseteq B$  iff  $B^c \subseteq A^c$