Problem Set #3

Due Wednesday, February 11, 2026 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the **Guidelines for Problem Sets** *on the course Webpage!*

1. Consider the statement $\exists x$ s.t. $x > x^3$.
   Give a domain where the statement is true and another where the statement is false. Explain.

2. For each statement,
   (i) Write it as an English sentence that does not use symbols for quantifiers;
   (ii) Write the negation in symbolic form;
   (iii) Write the negation as an English sentence that does not use symbols for quantifiers;
   (iv) Give the truth value of the statement and its negation (you do not need to prove these claims).
   (a) $\exists m \in \mathbb{Z}$ s.t. $\forall n \in \mathbb{Z}, m > n$
   (b) $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}$ s.t. $m^2 > n$

3. Are the following statements true or false? Give a proof for your conclusions.
   (a) If a, b, and c are integers, then $ab + ac$ is an even integer.
   (b) If b and c are odd integers and a is an integer, then $ab + ac$ is an even integer.

4. Prove that for all integers a and m, if a and m are the lengths of the legs of a right triangle with hypotenuse length $m + 1$, then a is an odd integer.
   Give examples of at least two different sets of integers a and m that satisfy the hypotheses of the claim.

References for problems: 2. Sundstrom, *Mathematical Reasoning Writing and Proof, Version 3*, Exercises 2.4.4 & 2.4.5; 3. Sundstrom, Exercise 1.2.7