1. For $p = 7$, compute $p^2 - 1$ and then divide by 24. What do you get?
   Repeat for $p = 13$ and $p = 41$. Now pick a large prime $p > 3$ and repeat.
   Do you think there's a general result?

2. Must there be at least two (non-bald) people in Boston with the same number of hairs on their heads?

3. Consider the sum $1 + 2 + 3 + \cdots + n$
   Compute this expression for $n = 4, 6, 10, 13, 100$.
   Look for a pattern for the sum that holds for every value of n.

4. You have a group of five people. Is it possible for each to be friends with exactly two others? What about with exactly three others?
   In a group of nine people, is it possible for each to be friends with exactly five others?

5. What is the minimum number of guests you need at a party to guarantee that either two people know each other or two people don't know each other?
   What about the same condition for three people? four people? five people?