Work on these with your partner(s) at the board

1. Define a sequence by $a_1 = 1, a_2 = 3,$ and $a_n = 3a_{n-1} - 2a_{n-2}\ \forall n \in \mathbb{N}, n \geq 3$
   (a) Compute the first six terms of the sequence, i.e, $a_1, \ldots, a_6$
   (b) Form a conjecture for the value of a_n that depends only on n
   (c) Use strong induction to prove your conjecture

Ernst, Exercise 4.27

2. Define the Fibonacci sequence by
   $f_0 = 0,\ f_1 = 1,$ and $f_n = f_{n-1} + f_{n-2}\ \ \forall n \in \mathbb{N}, n \geq 2$
   (a) Compute the first six terms of the sequence, i.e, $f_0, \ldots, f_5$
   (b) Use strong induction to prove that $\left(\frac{3}{2}\right)^{n-2} \leq f_n \leq 2^n \quad \forall n \in \mathbb{Z}, n \geq 1$
   (c) Let $\varphi = \frac{1+\sqrt{5}}{2}$ and $\psi = \frac{1-\sqrt{5}}{2}$. Prove that $f_n = \frac{\varphi^n - \psi^n}{\sqrt{5}} \quad \forall n \in \mathbb{Z}, n \geq 0$

Based on Ernst, Exercise 4.29