Work on these with your partner(s) at the board

1. Consider the sum $$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{n(n+1)}$$
   (a) Compute the sum for a few values of n

   (b) Form a conjecture for the value of the sum that depends only on n

   (c) Use induction to prove your conjecture

2. The *triangular numbers* are defined by: $t_1 = 1, \quad t_n = t_{n-1} + n \ \ \forall n \geq 2$
   (a) Prove that $8t_n + 1 = (2n+1)^2 \ \ \forall n \geq 1$

   (b) Compute $t_{n-1} + t_n$ for $n = 2, 3, 4, 5$

   Make a conjecture for the value of this sum for all $n \geq 2$, and then prove your conjecture

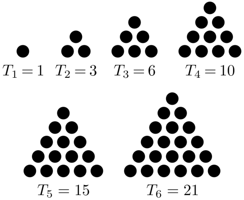

Image source: https://commons.wikimedia.org/wiki/File:First_six_triangular_numbers.svg

3. Determine whether each statement is true or false.
   If it is true, then give a proof. If it is false, then provide a counterexample.

   (a) $\forall n \in \mathbb{N},\ \ n^2 + n + 41$ is prime

   (b) $\forall n \in \mathbb{N},\ \ 6n^2 + 1$ is not a perfect square

   (c) $\forall n \in \mathbb{N},\ \ 23n^2 + 1$ is not a perfect square

   (d) $\forall n \in \mathbb{N},\ \ 991n^2 + 1$ is not a perfect square