Prove each of the following. Explicitly list the type of proof used.

1. $\forall a \in \mathbb{Q}, \ \exists b \in \mathbb{Q}$ s.t. $a^b \in \mathbb{Q}$

2. The sum of an even integer and an odd integer is an odd integer.

3. The square of an odd number is odd.

4. The sum any rational number and any irrational number is irrational.

5. $\forall n \in \mathbb{N}$ with $n \leq 4$, $(n+1)^3 \geq 3n$

6. The square of any odd integer has the form $8m + 1$ for some integer m.