Some Big Ideas, Week 3
Feb 2 – Feb 6, 2026

⊙ **Definition:** A **predicate** is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables.

The **domain** of a predicate variable is the set of all values that may be substituted in place of the variable.

If $P(x)$ is a predicate and x has domain D, the **truth set** of $P(x)$ is the set of all elements of D that make $P(x)$ true when they are substituted for x:

$$\{x \in D \mid P(x)\}$$

⊙ **Definition:** The **universal quantifier** $\forall$ means "for all":

$$\forall x \in D, P(x) \quad \text{means} \quad P(x) \text{ is true for every } x \in D$$

A universal statement is false if $P(x)$ is false for at least one $x \in D$.

⊙ **Definition:** The **existential quantifier** $\exists$ means "there exists":

$$\exists x \in D \text{ such that } P(x) \quad \text{means} \quad P(x) \text{ is true for at least one } x \in D$$

An existential statement is false if $P(x)$ is false for every $x \in D$.

⊙ The negation of a universal statement "$\forall x \in D, P(x)$" is logically equivalent to the statement "$\exists x \in D$ such that $\sim P(x)$".

⊙ The negation of an existential statement "$\exists x \in D$ such that $P(x)$" is logically equivalent to the statement "$\forall x \in D, \sim P(x)$".

Check the **Tentative Weekly Syllabus** on the course webpage for the specific sections relevant for this week.