Problem Set #4

Due Thursday, February 26, 2026 @ 11:59 pm
Submit as a single pdf file to Canvas

Remember to review the Guidelines for Problem Sets in the Course Policies on the course webpage when writing up your solutions. A list of computations with no explanation is not acceptable for the Problem Sets. The general rule of thumb is that you should give enough explanation so that you could hand your assignment to a student who took Calc II last semester and they could follow your thought process.

1. Show that the improper integral $\int_0^\infty x\, e^{-x}\, dx$ converges by finding the antiderivative and evaluating the limit.

   Hint: It may be helpful to review l'Hopital's rule for one part of your solution.

2. Use the Comparison and/or p-tests to determine if the improper integral converges or diverges.

   (a) $\int_1^\infty \frac{1}{x^5+5}\, dx$

   (b) $\int_{10}^\infty \frac{3}{\sqrt[3]{x}-2}\, dx$

3. Consider the series $\sum_{k=0}^{\infty} \left(\frac{3}{7}\right)^k$

   (a) What is the limit of the sequence of terms in the series? Why?
   (b) Use the Geometric Series Test to show that the series converges and find the value of the series.

4. Consider the series $\sum_{k=29}^{\infty} \frac{17}{6^k}$

   (a) Show that the series converges.
   (b) Find the exact value of the series.

   Be sure to justify your answers.