

Quick recap on Series

- ***n*th Term Test:** If $\lim_{k \rightarrow \infty} a_k \neq 0$, then $\sum_{k=0}^{\infty} a_k$ diverges

- **Geometric Series Test:** $\sum_{k=0}^{\infty} r^k$

- If $|r| \geq 1$, then the series diverges

- If $|r| < 1$, then the series converges to $\frac{1}{1-r}$

- **Integral Test:** If $a(x) > 0$, decreasing, and $a_k = a(k)$, then

$$\sum_{k=1}^{\infty} a_k \text{ and } \int_1^{\infty} a(x) dx$$

behave exactly the same. They either both converge or both diverge

Quick recap on Series (cont.)

- The p -test: $\sum_{k=1}^{\infty} \frac{1}{k^p}$
 - If $|p| \leq 1$, then the series diverges
 - If $p > 1$, then the series converges
- **Direct Comparison Test:** If $0 \leq a_k \leq b_k$ then $0 \leq \sum_{k=1}^{\infty} a_k \leq \sum_{k=1}^{\infty} b_k$ and
 - If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ also converges
 - If $\sum_{k=1}^{\infty} a_k$ diverges, then $\sum_{k=1}^{\infty} b_k$ also diverges
- **Alternating Series Test:** If $a_k \geq a_{k+1} > 0$ and $\lim_{k \rightarrow \infty} a_k = 0$, then $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ converges and S_n approximates the series within a_{n+1}

Quick recap on Series (cont.)

- Integral, p , and Direct Comparison tests apply only to positive series
- n th Term, Integral, p , and Direct Comparison determine convergence and/or divergence
 - They do *not* give limit of convergent series
 - Can approximate value by calculating a large partial sum, like S_{100}
- Alternating Series Test determines convergence and error bound when approximating value of convergent alternating series
- Geometric Series Test determines convergence, divergence, *and* exact value of convergent geometric series

Quick recap on Series (cont.)

- **Power Series:** A power series in x centered at c is a series

$$P(x) = a_0 + a_1(x - c) + a_2(x - c)^2 + a_3(x - c)^3 + \cdots = \sum_{k=0}^{\infty} a_k(x - c)^k$$

where each a_k is a constant.

- Big idea is that $P(x)$ is a *function*
For every value of x , $P(x)$ gives us a different series
- If $c = 0$ then the power series becomes

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots = \sum_{k=0}^{\infty} a_kx^k$$

Where we're going next

- If $f(x)$ is well-behaved at $x = 0$, then can we find coefficients to express $f(x)$ as a power series centered at $c = 0$:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots = \sum_{k=0}^{\infty} a_k x^k$$

- We've seen some evidence that

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

- Where did this come from?

How can we create series like this for other functions?