L_{10} will underestimate $\displaystyle\int_0^2 f(x)\, dx$
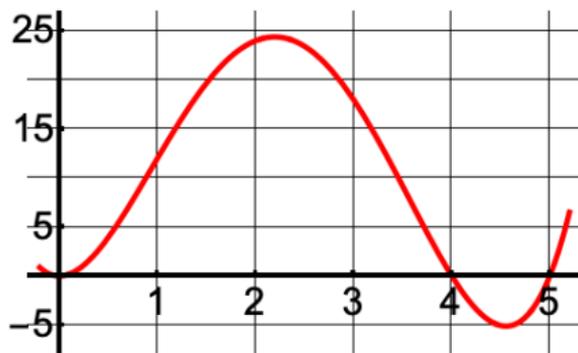
Plot of $y = f(x)$

(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Ummmm . . .

R_{10} will overestimate $\displaystyle\int_3^4 f(x)\, dx$
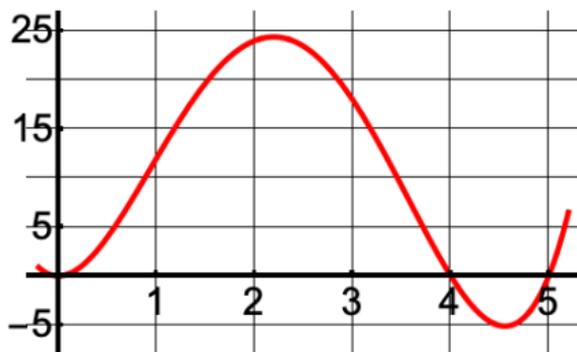

Plot of $y = f(x)$

(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Ummmm . . .

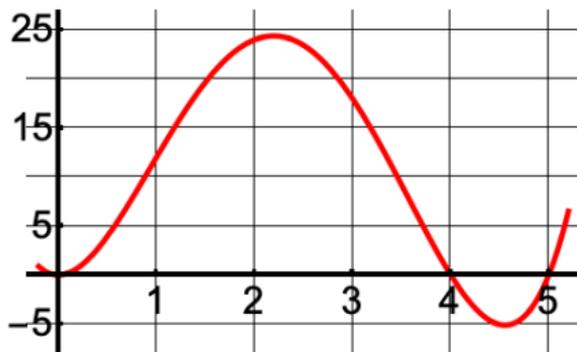T_{10} will overestimate $\displaystyle\int_1^3 f(x)\,dx$

Plot of $y = f(x)$

(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Ummmm ...

Example: Let $\mathcal{I} = \displaystyle\int_0^4 \sin(x^2) + 3 \ dx$
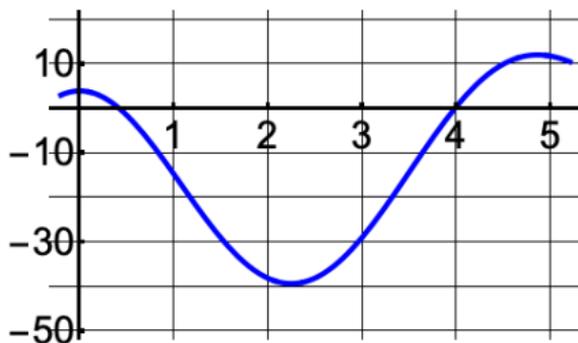
1. Calculate T_{15} and S_{15} to approximate $\mathcal{I}$

   Useful WolframAlpha syntax:
   ```
   trapezoidal rule of sin(x^2)+3 from x=0 to x=4 with 15 subdivisions
   ```

Using Theorem 5.5.1, $M_2 = 20$ is a valid value for $\displaystyle\int_4^5 f(x)\, dx$

(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Ummmm . . .

Plot of $y = f''(x)$

Using Theorem 5.5.1, $M_2 = 20$
is a valid value for $\displaystyle\int_1^5 f(x)\,dx$
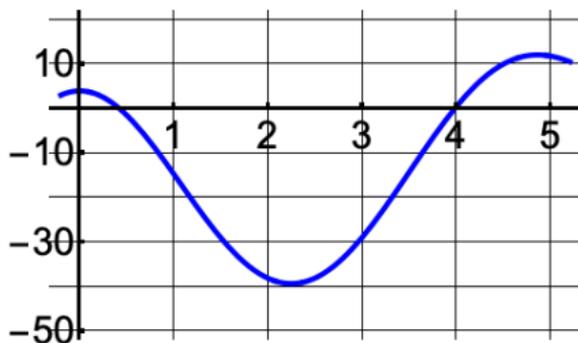
(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Ummmm . . .

Plot of $y = f''(x)$

Example: Let $\mathcal{I} = \displaystyle\int_0^4 \sin(x^2) + 3 \ dx$

1. Calculate T_{15} and S_{15} to approximate $\mathcal{I}$

   Useful WolframAlpha syntax:
   ```
   trapezoidal rule of sin(x^2)+3 from x=0 to x=4 with 15 subdivisions
   ```

2. What is the error from using the trapezoidal rule?

   Useful Desmos syntax:
   ```
   | d/dx  d/dx  sin(x^2) + 3| {0 <= x <= 4}
   ```

3. What is the error from using Simpson's rule?

Let $\mathcal{I} = \displaystyle\int_5^{10} \cos\left(\dfrac{x^2}{3}\right) + x \ dx$

1. Calculate T_{30} and use Theorem 5.5.1 to determine the size of the error $|\mathcal{I} - T_{30}|$

2. Calculate S_{30} and use Theorem 5.5.1 to determine the size of the error $|\mathcal{I} - S_{30}|$