If p is prime, how do we find $\alpha \in \mathbb{F}_p^*$ of large prime order?

Exercise 1.33 gives a reliable way to find such an α in general.
This problem looks at a specific case.

1. Use Mathematica to verify that $p = 415\,643$ is prime.

2. Verify that $q = 207\,821$ is prime and that $q \mid (p - 1)$. What is $\frac{p-1}{q}$?

3. What are all of the possible orders of elements in $\mathbb{F}_p^*$?

4. Let $a = 6$, and compute $a^2 \mod p$ and $a^q \mod p$.
   Use your answers to determine $\text{ord}(a)$ in $\mathbb{F}_p^*$.

5. Repeat (d) with $a = 9$ and $a = 415642$.

6. Find $\text{ord}(a^2)$ in $\mathbb{F}_p^*$ for the following values of a:

$$a = 6, \quad a = 9, \quad a = 100000, \quad a = 415642$$

   Pick a few more random values of a and find $\text{ord}(a^2)$ in $\mathbb{F}_p^*$.