Gram-Schmidt Algorithm

Suppose $\mathcal{B} = \left\{\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}, \ldots, \vec{\mathbf{v_n}}\right\}$ is the basis for an subspace $S \subset \mathbb{R}^m$.

Form the set $\mathcal{B}^* = \left\{\vec{\mathbf{v_1}}^*, \vec{\mathbf{v_2}}^*, \ldots, \vec{\mathbf{v_n}}^*\right\}$ by

$$\vec{\mathbf{v_1}}^* = \vec{\mathbf{v_1}}$$

$$\vec{\mathbf{v_2}}^* = \vec{\mathbf{v_2}} - \frac{\vec{\mathbf{v_2}} \cdot \vec{\mathbf{v_1}}^*}{\vec{\mathbf{v_1}}^* \cdot \vec{\mathbf{v_1}}^*}\vec{\mathbf{v_1}}^*$$

$$\vec{\mathbf{v_3}}^* = \vec{\mathbf{v_3}} - \frac{\vec{\mathbf{v_3}} \cdot \vec{\mathbf{v_2}}^*}{\vec{\mathbf{v_2}}^* \cdot \vec{\mathbf{v_2}}^*}\vec{\mathbf{v_2}}^* - \frac{\vec{\mathbf{v_3}} \cdot \vec{\mathbf{v_1}}^*}{\vec{\mathbf{v_1}}^* \cdot \vec{\mathbf{v_1}}^*}\vec{\mathbf{v_1}}^*$$

$$\vdots$$

$$\vec{\mathbf{v_i}}^* = \vec{\mathbf{v_i}} - \sum_{j=1}^{i-1} \mu_{i,j}\vec{\mathbf{v_j}}^* \qquad \text{where } \mu_{i,j} = \frac{\vec{\mathbf{v_i}} \cdot \vec{\mathbf{v_j}}^*}{\vec{\mathbf{v_j}}^* \cdot \vec{\mathbf{v_j}}^*}, \quad 1 \le j < i$$

Then $\mathcal{B}^*$ is an orthogonal basis for S.