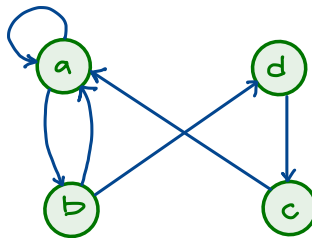


PROBLEM SET #8

Due Thursday, April 10, 2025 @ 11:59 pm
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course Webpage!*

1. Let S be the relation defined on $\mathbb{R} \times \mathbb{R}$ by $(x, y) S (a, b)$ iff $x^2 + y^2 = a^2 + b^2$.
Prove that S is an equivalence relation and describe the equivalence classes geometrically.
2. Let T be the relation defined on $\mathbb{R} \times \mathbb{R}$ by $(x, y) T (a, b)$ iff $3x + 2y = 3a + 2b$.
Prove that T is an equivalence relation and describe the equivalence classes geometrically.
3. We can visualize a relation R on a set A by drawing a *directed graph*, or *digraph*, where the vertices are the elements of A and we draw an edge from vertex a to vertex b if $(a, b) \in R$.
For example, $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, a), (b, d), (c, a), (d, c)\}$ then the digraph is



Let $A = \{1, 2, 3, 4, 5, 6\}$ and define

$$R = \{(1, 1), (1, 6), (2, 2), (2, 3), (2, 4), (3, 3), (3, 2), (3, 4), (4, 4), (4, 2), (4, 3), (5, 5), (6, 6), (6, 1)\}$$

- (a) Draw the digraph for R .
- (b) Determine whether R is an equivalence relation on A .
- (c) In general, if S is an equivalence relation on a set B , describe the digraph for S .
Be sure to include a description of the equivalence classes of S .
4. (a) What is the probability that a randomly chosen integer from 500 through 699 is even?
(b) What is the probability that a randomly chosen integer from 500 through 699 is odd?
(c) What is the probability that a randomly chosen integer from 1400 through 1799 is a multiple of 7?
(d) How many odd integers between 1000 and 5999 have distinct digits?

References for problems: 1,2. Schumacher, *Chapter Zero*, Exercise 4.3.19; 3. Ernst, *Introduction to Proof via Inquiry-Based Learning*, Exercise 7.36