Problem Set #7

Due Thursday, April 3, 2025 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the **Guidelines for Problem Sets** *on the course Webpage!*

1. Let $S = [0,1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$ and $T = [0,2] = \{x \in \mathbb{R} \mid 0 \leq x \leq 2\}$

   (a) Prove that S and T have the same cardinality.

   (b) Compare S and T. What unintuitive (at least to me!) property of infinite sets does your proof from part (a) demonstrate?

2. If $f : X \to Y$ and $g : Y \to Z$ are functions and $g \circ f$ is one-one, must f be one-one?
   Prove or give a counterexample.

3. Define a relation R on $\mathbb{R}$ by $x\ R\ y$ iff $xy > 0$.

   (a) Give three ordered pairs in R.

   (b) Give three ordered pairs of real numbers not in R.

   (c) Prove that R is symmetric and transitive but not reflexive.

4. Define a relation P on $\mathbb{N}$ by $n\ P\ m$ iff $\exists$ a prime number p s.t. $p|n$ and $p|m$.

   (a) Give three ordered pairs in P.

   (b) Give three ordered pairs of natural numbers not in P.

   (c) Is P symmetric? reflexive? transitive? Prove your claims.

References for problems: 3, 4 Epp, *Discrete Mathematics with Applications, 4th edition*, Exercises 8.2.9, 17