

PROBLEM SET #6

Due Thursday, March 27, 2025 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the *Guidelines for Problem Sets* on the course Webpage!

1. Prove that $\sqrt{2}$ is irrational.

Hint: Consider a proof by contradiction.

2. Provide a proof for each statement that is true, and find a counter-example for each statement that is false. Assume that all sets are subsets of a universal set U .

(a) For all sets A and B , if $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

(b) For all sets A and B , $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

(c) For all sets A, B and C , if $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

3. Define $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$ by $f(x, y) = (x + 7, 2 - 3y)$

(a) What is $f(1, 2)$? $f^{-1}(9, 5)$?

(b) Is f one-one? Prove your claim.

(c) Is f onto? Prove your claim.

4. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are functions and $g \circ f$ is onto, must f be onto? Prove or give a counterexample.