Problem Set #5

Due Thursday, March 20, 2025 @ 11:59 pm
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course Webpage!*

1. Consider the sequence defined by

$$a_0 = 12, \quad a_1 = 29, \quad \text{and} \quad a_k = 5a_{k-1} - 6a_{k-2}, \ \forall k \in \mathbb{Z}, k \geq 2$$

   (a) Compute the first six terms of the sequence (through a_5).
   (b) Prove that $a_n = 5 \cdot 3^n + 7 \cdot 2^n \quad \forall n \in \mathbb{Z}, n \geq 0$.

2. Define the Fibonacci sequence by $f_0 = 0, \ f_1 = 1, \ \text{and} \ f_n = f_{n-1} + f_{n-2} \ \ \forall n \in \mathbb{N}, n \geq 2$

   Let $\varphi = \dfrac{1+\sqrt{5}}{2}$ and $\psi = \dfrac{1-\sqrt{5}}{2}$.

   Prove that $f_n = \dfrac{\varphi^n - \psi^n}{\sqrt{5}} \quad \forall n \in \mathbb{Z}, n \geq 0$

   Hint: It may be useful to observe that $\varphi^2 = \dfrac{3+\sqrt{5}}{2}$ and $\psi^2 = \dfrac{3-\sqrt{5}}{2}$.

3. If A and C are sets, prove that $A = (A \cap C) \cup (A - C)$

4. Assume A and B are all subsets of some universal set U. Prove $A \subseteq B$ iff $B^c \subseteq A^c$

References for problems: 1. Epp, Exercises 5.4.5;