

PROBLEM SET #3

Due Thursday, February 13, 2025 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the *Guidelines for Problem Sets* on the course Webpage!

1. Consider the statement $\exists x$ s.t. $x > x^3$.
Give a domain where the statement is true and another where the statement is false. Explain.

2. For each statement,
 - (i) Write it as an English sentence that does not use symbols for quantifiers;
 - (ii) Write the negation in symbolic form;
 - (iii) Write the negation as an English sentence that does not use symbols for quantifiers;
 - (iv) Give the truth value of the statement and its negation (you do not need to prove these claims).
 - (a) $\exists m \in \mathbb{Z}$ s.t. $\forall n \in \mathbb{Z}, m > n$
 - (b) $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}$ s.t. $m^2 > n$

3. Are the following statements true or false? Give a proof for your conclusions.
 - (a) If a, b , and c are integers, then $ab + ac$ is an even integer.
 - (b) If b and c are odd integers and a is an integer, then $ab + ac$ is an even integer.

4. Prove that for all integers a and m , if a and m are the lengths of the legs of a right triangle with hypotenuse length $m + 1$, then a is an odd integer.
Give examples of at least two different sets of integers a and m that satisfy the hypotheses of the claim.