Problem Set #2

Due Thursday, February 6, 2025 @ 11:59 pm
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course Webpage!*

1. (a) Use a truth table to show that $p \to q \ \equiv \ (\sim p \vee q)$

    (b) Use DeMorgan's Law and part (a) to simplify $\sim (p \to q)$

    (c) Use DeMorgan's Law and part (a) to simplify $\sim \big((\sim (p \wedge q)) \to (r \wedge s)\big)$

2. (a) Show that the following statements are all logically equivalent

    $$p \to (q \vee r), \qquad (p \wedge \sim q) \to r, \qquad (p \wedge \sim r) \to q$$

    (b) Use the logical equivalences from part (a) to rewrite the following sentence in two different ways.
    If n is prime, then n is odd or $n = 2$.

3. Legal Seafoods restaurants use the tagline "If it isn't fresh, it isn't Legal". This is true, by the way.

    (a) Write the converse of the tagline. Do you think this is true?

    (b) Write the contrapositive of the tagline. Do you think this is true?

    (c) Use 1(b) to write the negation of the tagline.

    (d) Suggest a logically equivalent tagline that avoids the use of "isn't" and "not".

4. Let the domain of x be $\mathbb{R}$, and let

    $\mathcal{I}(x)$ be "x is an integer",
    $\mathcal{N}(x)$ be "x is a natural number", and
    $\mathcal{P}(x)$ be "x is positive"

    Rewrite each statement as an English sentence without using quantifiers or variables. Indicate which statements are true and which are false. Be sure to explain your answers.

    (a) $\forall x, \mathcal{I}(x) \to \mathcal{N}(x)$

    (b) $\forall x, (\mathcal{I}(x) \wedge \mathcal{P}(x)) \to \mathcal{N}(x)$

    (c) $\exists x$ such that $\mathcal{I}(x) \wedge \sim \mathcal{N}(x)$