Work on these with your partner(s) at the board

1. Let $X = \{1, 2, 3, 4\}$, $Y = \{A, B, C\}$, and $Z = \{\circ, \triangle, \square, \heartsuit\}$
   Give an example of each type of function
   (a) $f : X \to Y$ is onto but not one-one
   (b) $g : Y \to Z$ is one-one but not onto
   (c) $h : Z \to X$ is one-one and onto
   (d) $k : Y \to X$ is neither one-one nor onto

2. Define $m : \mathbb{Z} \to \{0, 1, 2, 3, 4\}$ by $m(n) = r$ where r is the remainder when n is divided by 5.
   (a) What is $m(6)$? $m(10)$? $m(-3)$? $m(3)$?
   (b) What is $m(\{7, 8\})$?
   (c) What is $m^{-1}(0)$? $m^{-1}(1)$? $m^{-1}(\{2, 3\})$?

   Note: It may help to review the quotient remainder property from Week 4.

5. Let S be the set of all strings of 0's and 1's of length 3 (e.g. $000 \in S, 101 \in S$), and let $A = \{a, b, c\}$.
   (a) List the elements of S
   (b) Define a bijection $f : S \to \mathcal{P}(A)$
6. Let T be the set of all strings of 0's and 1's of length n, and let $B = \{b_1, b_2, \cdots, b_n\}$.
   (a) What is $|T|$?
   (b) Define a bijection $g : T \to \mathcal{P}(B)$
7. Give an example of a function $f : \mathbb{N} \to \mathbb{Z}$ that is one-one but not onto
8. Give an example of a function $g : \mathbb{Z} \to \mathbb{Z}$ that is one-one but not onto
9. Let E be the set of even natural numbers. Give an example of a bijection $h : \mathbb{N} \to E$
10. Give an example of a bijection $k : \mathbb{N} \to \mathbb{Z}$