Work on these with your partner(s) at the board

1. Let $P(x)$ be the predicate "$x^2 \geq x$"
   (a) What are the truth values of $P(2)$? $P(\frac{1}{2})$?, $P(-1)$?
   (b) If the domain is $D = \mathbb{Z}$, find the truth set of $P(x)$
   (c) If the domain is $D = \mathbb{R}$, find the truth set of $P(x)$

2. Let $Q(x)$ be the predicate "$x^4 \geq x$". Determine the truth value of each statement.
   (a) $\forall x \in \mathbb{Z}, Q(x)$
   (b) $\forall x \in \mathbb{R}, Q(x)$
   (c) $\exists x \in \mathbb{R}$ such that $Q(x)$

3. Rewrite the following informally as English sentences without quantifiers or variables:
   (a) $\forall x \in \mathbb{Z}$, if $x > 0$, then $x^2 > 0$
   (b) $\exists x \in \mathbb{R}$ such that $x^2 = 9$

4. Let $\mathbb{R}$ be the domain of the predicate variable x. Which of the following are true and which are false? Give counter examples for those that are false.
   (a) $x > 2 \;\Rightarrow\; x^2 > 4$
   (b) $x^2 > 4 \;\Rightarrow\; x > 2$
   (c) $x^2 > 4 \;\Leftrightarrow\; |x| > 2$

   Epp, Exercise 3.22

5. Determine the truth value of each statement.
   (a) $\exists a, b, c \in \mathbb{Z},$ all non-zero, such that $a^2 + b^2 = c^2$
   (b) $\exists a, b, c \in \mathbb{Z},$ all non-zero, such that $a^3 + b^3 = c^3$

6. Rewrite the following statements formally as a predicate with a quantifier. Be certain to define the domain. Then write formal and informal negations.
   (a) There is a professor at Wheaton who went to graduate school at Northwestern.
   (b) All customers must wear shoes.

7. Consider the statement: $\forall\, x \in \mathbb{R},$ if $x^2 > 9$ then $x > 3$ or $x < -3$

   Write the negation,

   the converse,

   the inverse, and

   the contrapositive.

   Indicate which are true and which are false.

8. Determine the truth of each of the following.
   (a) $\forall\, x \in \mathbb{R}, \forall\, y \in \mathbb{R}, x + y = y + x$
   (b) $\exists\, x \in \mathbb{N}, \exists\, y \in \mathbb{N}$ such that $x + y = 5$
   (c) $\forall\, x \in \mathbb{R}, \forall\, y \in \mathbb{R}, x + y = 5$
   (d) $\forall\, x \in \mathbb{Z}, \exists\, y \in \mathbb{Z}$ such that $x + y = 5$
   (e) $\exists\, x \in \mathbb{Z}$ such that $\forall\, y \in \mathbb{Z}, x + y = y$

9. For each of the following, rewrite the statement in English without $\forall$ or $\exists$ or variables, as simply as possible. Then write the negation.
   (a) $\exists$ a book b such that $\forall$ people p, p has read b
   (b) $\forall\, x \in \mathbb{N}, \exists\, y \in \mathbb{Q}$ such that $x \cdot y = 1$

   Epp, Exercise 3.3.14