# Work on these with your partner(s) at the board

1. Prove that the third Ramsey number $R(3) > 5$ by two-coloring the edges of $K_5$ such that there is no blue $K_3$ or red $K_3$ that is a subgraph.
2. Prove that $R(3) = 6$ by showing that every two-coloring of the edges of $K_6$ has a $K_3$ subgraph that is red or blue.
3. Prove that $R(4) > 6$
4. The purpose of this problem is to show that $R(4) > 17$.
   Label the vertices of $K_{17}$ by $0, 1, 2, \ldots, 16$
   Color an edge connecting vertex $i$ to vertex $j$ red iff

   $$(i - j) \mod 17 \equiv 1, 2, 4, 8, 9, 13, 15, \text{ or } 16$$

   Otherwise, color the edge blue.
   (a) Prove that there is no $K_4$ subgraph with red edges that contains vertex 0.
   (b) Prove that there is no $K_4$ subgraph with blue edges that contains vertex 0.
   (c) Argue by symmetry that the same argument works for all other vertices.

   Conclude that $R(4) > 17$.