

## Some Big Ideas, Week 5

Feb 17 – Feb 21, 2025

⊙ ***Principle of Mathematical Induction:***

Let  $P(n)$  be a property that is defined for integers  $n$ . Let  $a$  be a fixed integer.

Suppose the following two statements are true:

1.  $P(a)$  is true.
2.  $\forall k \geq a$ , if  $P(k)$  is true, then  $P(k + 1)$  is true.

Then  $\forall n \geq a$ ,  $P(n)$  is true.

⊙ ***General structure of a Proof by Induction:***

Start by giving the statement that you want to prove:

*Let  $P(n)$  be the statement ...*

To prove  $P(n)$  is true for all  $n \geq a$ , requires two steps:

1. ***Base case:*** Prove that  $P(a)$  is true.
2. ***Inductive case:*** Assume that  $P(k)$  is true, and prove that  $P(k + 1)$  is true.  
“ $P(k)$  is true” is called the ***inductive hypothesis***.

If you successfully proved both results, then you can conclude

*Thus, by the principle of mathematical induction,  $P(n)$  is true  $\forall n \geq a$ .*

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Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 4th edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*.

Check the ***Tentative Weekly Syllabus*** on the course webpage for the specific sections relevant for this week.