# Some Big Ideas, Week 4

## Feb 10 – Feb 14, 2025

⊙ Some facts about numbers that we'll assume are true:

- The basic properties of the algebra of real numbers hold, including the commutative, associative, and distributive laws, etc.
  cf. Sundstrom, Table 1.2 pg 18; Ernst, Axioms 5.1, pg 61; Epp, Appendix A
- The set of integers is closed under addition, subtraction, and multiplication.
  Note that the integers are *not* closed under division since the quotient of two integers need not be an integer.
- **Definition:** A number $n$ is ***even*** if $\exists\ k \in \mathbb{Z}$ such that $n = 2k$.
- **Definition:** A number $m$ is ***odd*** if $\exists\ j \in \mathbb{Z}$ such that $m = 2j + 1$.
- **Definition:** An integer $p > 1$ is ***prime*** iff the only natural numbers that evenly divide $p$ are 1 and itself. A number that is not prime is called ***composite***.
- Given any $n \in \mathbb{Z}$ and any $d \in \mathbb{N}$, we can write $n = qd + r$ where $q, r \in \mathbb{Z}$ with $0 \leq r < d$.

⊙ A direct proof for a statement "$\forall\ x \in D,$ if $P(x),$ then $Q(x)$" will usually have the form:

Let $x \in D$ be an arbitrary value in the domain where $P(x)$ is true.

*Argument, argument, argument.*

Conclude that $Q(x)$ is true.

⊙ To disprove a statement "$\forall\ x \in D,$ if $P(x),$ then $Q(x)$" by counterexample, find a specific $x \in D$ such that $P(x)$ is true and $Q(x)$ is false.

⊙ To prove a statement by contradiction:

Assume the negation of the statement is true.

Show that this logically leads to a contradiction.

Conclude that the statement is true.

⊙ To prove a statement "$P(x) \to Q(x)$" by contraposition:

Assume $\sim Q(x)$ is true.

*Argument, argument, argument.*

Conclude that $\sim P(x)$ is true.

Thus $\sim Q(x) \to \sim P(x)$, the contrapositive of the statement to be proved.

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Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 4th edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*.

Check the ***Tentative Weekly Syllabus*** on the course webpage for the specific sections relevant for this week.