

## Some Big Ideas, Week 11

Apr 7 – Apr 11, 2025

- ⊙ **Definition:** A *sample space* is the set of all possible outcomes of an experiment or random process.

An *event* is a subset of a sample space.

- ⊙ **Definition:** If  $S$  is a finite sample space where all outcomes are equally likely and  $E$  is an event in  $S$ , then the *probability of  $E$*  is

$$P(E) = \frac{\text{number of outcomes in } E}{\text{total number of outcomes in } S} = \frac{|E|}{|S|}$$

*Note:* Epp uses the notation  $N(A)$  for the cardinality of a finite set rather than  $|A|$ .

- ⊙ **Additive Principle:** If an event  $E$  can occur in  $m$  ways and the event  $F$  can occur in  $n$  ways where  $E$  and  $F$  are disjoint, then the event “ $E$  or  $F$ ” can occur in  $m + n$  ways.

- ⊙ **Multiplicative Principle:** If an event  $E$  can occur in  $m$  ways, and each possibility for  $E$  allows for exactly  $n$  ways for the event  $F$  to occur, then the event “ $E$  and  $F$ ” can occur in  $m \cdot n$  ways.

- ⊙ **Definition:** A *permutation* of a set  $A$  is an ordered, non-repetitive arrangement of all elements of  $A$ .

For example, if  $A = \{a, b, c\}$ , then there are six permutations of  $A$ :  $abc, acb, cab, cba, bca, bac$

- ⊙ **Definition:** A  *$k$ -permutation* of  $A$  an ordered, non-repetitive arrangement of  $k$  elements of  $A$ .

For example, if  $A = \{a, b, c, d\}$ , a 2-permutation of  $A$  is  $bc$  and a 3-permutation is  $cad$ .

- ⊙ **Theorem (Epp 9.2.3):** The number of  $k$ -permutations from a set with  $n$  elements, denoted  $P(n, k)$ , is

$$P(n, k) = \frac{n!}{(n - k)!}$$

- ⊙ If  $A$  and  $B$  are finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 4th edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*.

Check the *Tentative Weekly Syllabus* on the course webpage for the specific sections relevant for this week.