Problem Set #7

Due Thursday, April 4, 2024 @ 11:59 pm
Submit as single pdf file to Canvas

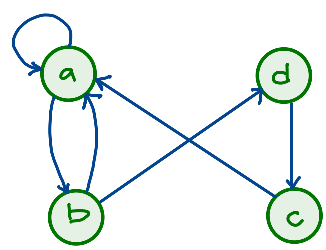
Remember to review the Guidelines for Problem Sets on the course webpage.

1. Define a relation R on $\mathbb{R}$ by $x \; R \; y$ iff $xy > 0$.
   (a) Give three ordered pairs in R.
   (b) Give three ordered pairs of real numbers not in R.
   (c) Prove that R is symmetric and transitive but not reflexive.

2. Let S be the relation defined on $\mathbb{R} \times \mathbb{R}$ by $(x, y) \; S \; (a, b)$ iff $x^2 + y^2 = a^2 + b^2$.
   Prove that S is an equivalence relation and describe the equivalence classes.

3. Let T be the relation defined on $\mathbb{R} \times \mathbb{R}$ by $(x, y) \; T \; (a, b)$ iff $3x + 2y = 3a + 2b$.
   Prove that T is an equivalence relation and describe the equivalence classes.

4. We can visualize a relation R on a set A by drawing a *directed graph*, or *digraph*, where the vertices are the elements of A and we draw an edge from vertex a to vertex b if $(a, b) \in R$.
   For example, $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, a), (b, d), (c, a), (d, c)\}$ then the digraph is

   Let $A = \{1, 2, 3, 4, 5, 6\}$ and define

   $$R = \{(1,1), (1,6), (2,2), (2,3), (2,4), (3,3), (3,2), (3,4), (4,4), (4,2), (4,3), (5,5), (6,6), (6,1)\}$$

   (a) Draw the digraph for R.
   (b) Determine whether R is an equivalence relation on A.
   (c) In general, if S is an equivalence relation on a set B, describe the digraph for S.
   Pay special attention to the equivalence classes of S.

References for problems: 2,3. Schumacher, *Chapter Zero*, Exercise 4.3.19; 4. Ernst, *Introduction to Proof via Inquiry-Based Learning*, Exercise 7.36