Problem Set #6

Due Thursday, March 28, 2024 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Provide a proof for each statement that is true, and find a counter-example for each statement that is false. Assume that all sets are subsets of a universal set U.
   (a) For all sets A and B, if $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
   (b) For all sets A and B, $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.
   (c) For all sets A and B, $\mathcal{P}(A \times B) \subseteq \mathcal{P}(A) \times \mathcal{P}(B)$.

2. Prove or give a counterexample: For all sets A, B and C, if $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

3. Define $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ by $f(x, y) = (x + 7, 2 - 3y)$
   (a) Is f one-one? Prove or give a counterexample.
   (b) Is f onto? Prove or give a counterexample.

4. If $f : X \to Y$ and $g : Y \to Z$ are functions and $g \circ f$ is onto, must f be onto? Prove or give a counterexample.

References for problems: 1a,b,c. Epp, *Discrete Mathematics with Applications, 4th edition*, Exercises 6.3.17, 18, 21; 4. Epp, Exercise 7.3.17