Problem Set #5

Due Thursday, February 29, 2024 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Let $x \in \mathbb{R}$. Prove x is irrational iff $\dfrac{1}{x}$ is irrational.

2. Prove $3 \mid (n^3 - 7n + 3) \quad \forall$ integers $n \geq 0$.

3. Prove $\sqrt{n} < 1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \cdots + \dfrac{1}{\sqrt{n}} \quad \forall n \in \mathbb{Z}, n \geq 2.$

   Hint: At some point in your proof, it might be useful to remember that $\sqrt{k} < \sqrt{k+1} \quad \forall k \in \mathbb{N}$.

4. Consider the sequence defined by

$$a_0 = 12, \quad a_1 = 29, \quad \text{and} \quad a_k = 5a_{k-1} - 6a_{k-2}, \ \forall k \in \mathbb{Z}, k \geq 2$$

   (a) Compute the first six terms of the sequence (through a_5).

   (b) Prove that $a_n = 5 \cdot 3^n + 7 \cdot 2^n \quad \forall n \in \mathbb{Z}, n \geq 0$.

References for problems: 2. Epp, *Discrete Mathematics with Applications, 4th edition*, Exercise 5.3.10; 3. Epp, Exercise 5.3.21; 4. Epp, Exercises 5.4.5