Problem Set #3

Due Thursday, February 15, 2024 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Consider the statement $\exists x$ s.t. $x > x^3$.
   Give a domain where the statement is true and another where the statement is false. Explain.

2. For each statement,
   (i) Write it as an English sentence that does not use symbols for quantifiers;
   (ii) Write the negation in symbolic form;
   (iii) Write the negation as an English sentence that does not use symbols for quantifiers;
   (iv) Give the truth value of the statement and its negation (you do not need to prove these claims).

   (a) $\exists m \in \mathbb{Z}$ s.t. $\forall n \in \mathbb{Z}, m > n$

   (b) $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}$ s.t. $m^2 > n$

3. Consider $\forall y\ \exists x \exists z$ s.t. $T(x, y, z) \wedge Q(x, y)$

   (a) Suppose that
   the domain of x is the set of math classes offered at Wheaton,
   the domain of y is the set of students in Discrete Math, and
   the domain of z is the set of all math professors at Wheaton.
   Let $T(x, y, z)$ be the predicate "y took class x with z" and
   $Q(x, y)$ be the predicate "y loves the class x".
   What does the original statement say in words?

   (b) Negate the statement so that all negation symbols immediately precede predicates.
   Be sure to show all your steps!

   (c) Using the same definitions as in (a), what does the negation of the statement mean in words?

4. Are the following statements true or false? Give a proof for your conclusions.

   (a) If a, b, and c are integers, then $ab + ac$ is an even integer.

   (b) If b and c are odd integers and a is an integer, then $ab + ac$ is an even integer.

5. Prove that for all integers a and m, if a and m are the legs of a right triangle with hypotenuse $m + 1$, then a is an odd integer.

   Give examples of at least two different sets of integers a and m that satisfy the hypotheses of the claim.

References for problems: 1,3. Rachelle DeCoste; 2. Sundstrom, *Mathematical Reasoning Writing and Proof, Version 3*, Exercises 2.4.4 & 2.4.5; 4. Sundstrom, Exercise 1.2.7