Problem Set #2

Due Thursday, February 8, 2024 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage.

1. (a) Use a truth table to show that $p \to q \equiv (\sim p \vee q)$

   (b) Use DeMorgan's Law and part (a) to simplify $\sim (p \to q)$

   (c) Use DeMorgan's Law and part (a) to simplify $\sim \big((\sim (p \wedge q)) \to (r \wedge s)\big)$

2. (a) Show that the following statements are all logically equivalent

$$p \to (q \vee r), \qquad (p \wedge \sim q) \to r, \qquad (p \wedge \sim r) \to q$$

   (b) Use the logical equivalences from part (a) to rewrite the following sentence in two different ways.
   If n is prime, then n is odd or $n = 2$.

3. Legal Seafoods restaurants use the tagline "If it isn't fresh, it isn't Legal".

   (a) Write the converse of the tagline. Do you think this is true?

   (b) Write the contrapositive of the tagline. Do you think this is true?

   (c) Use 1(b) to write the negation of the tagline.

   (d) Suggest a logically equivalent tagline that avoids the use of "isn't" and "not".

4. Let $Q(n)$ be the predicate "$n^2 \leq 30$"

   (a) Write $Q(2), Q(-2), Q(7)$, and $Q(-7)$, and indicate which of these statements are true, and which are false.

   (b) Find the truth set of $Q(n)$ if the domain of n is $\mathbb{Z}$.

   (c) Find the truth set of $Q(n)$ if the domain of n is $\mathbb{N}$.

5. Let the domain of x be $\mathbb{R}$, and let

   $\mathcal{I}(x)$ be "x is an integer",
   $\mathcal{N}(x)$ be "x is a natural number", and
   $\mathcal{P}(x)$ be "x is positive"

   Rewrite each statement as an English sentence without using quantifiers or variables. Indicate which statements are true and which are false. Be sure to explain your answers.

   (a) $\forall x, \mathcal{I}(x) \to \mathcal{N}(x)$

   (b) $\forall x, (\mathcal{I}(x) \wedge \mathcal{P}(x)) \to \mathcal{N}(x)$

   (c) $\exists x$ such that $\mathcal{I}(x) \wedge \sim \mathcal{N}(x)$

References for problems: 2. Epp, *Discrete Mathematics with Applications, 4th edition*, Exercise 2.2.14; 4. Epp, Exercise 3.1.4; 5. Epp, Exercise 3.1.28