Work on these with your partner(s) at the board

1. Let X and Y be finite sets and $f : X \to Y$ a function.
   (a) Give an example where f is one-one but not onto
   (b) Give an example where f is onto but not one-one
   (c) Give an example where f is both one-one and onto

   Note: You can change X and Y for each part.

2. Repeat #1 with $X = \mathbb{R}$ and $Y = \mathbb{R}$

3. Repeat #1 with $X = \mathbb{Z}$ and $Y = \mathbb{Z}$

4. Define $m : \mathbb{Z} \to \{0, 1, 2, 3, 4\}$ by $m(n) = r$ where r is the remainder when n is divided by 5.
   (a) What is $m(6)$? $m(10)$? $m(-3)$? $m(3)$?
   (b) What is $m(\{7, 8\})$?
   (c) What is $m^{-1}(0)$? $m^{-1}(1)$? $m^{-1}(\{2, 3\})$?

   Note: It may help to review the quotient remainder property from Week 4.

5. Find the domain and range of each of the following functions:
   (a) The function that assigns to each natural number its last digit (in its base 10 representation)
   (b) The function that assigns to a finite string of 0's and 1's the number of 1's in the string.

   From Rachelle DeCoste

6. Let S be the set of all strings of 0's and 1's of length 3, and let $A = \{a, b, c\}$.
   (a) List the elements of S
   (b) Define a bijection $f : S \to \mathcal{P}(A)$

7. Let T be the set of all strings of 0's and 1's of length n, and let $B = \{b_1, b_2, \cdots, b_n\}$.
   (a) What is $|T|$?
   (b) Define a bijection $g : T \to \mathcal{P}(B)$

8. Repeat #1 with $X = \mathbb{Z}$ and $Y = \mathbb{N}$

9. Repeat #1 with $X = \mathbb{N}$ and $Y = \mathbb{N} \times \mathbb{N}$