

## Discuss these with your partner(s)

1. For  $p = 7$ , compute  $p^2 - 1$  and then divide by 24. What do you get?  
Repeat for  $p = 13$  and  $p = 41$ . Now pick a large prime  $p > 3$  and repeat.  
Do you think there's a general result?
2. Must there be at least two (non-bald) people in Boston with the same number of hairs on their heads?
3. Consider the sum  $1 + 2 + 3 + \cdots + n$   
Compute this expression for  $n = 4, 6, 10, 13, 100$ .  
Look for a pattern for the sum that holds for every value of  $n$ .
4. You have a group of five people. Is it possible for each to be friends with exactly two others? What about with exactly three others?  
In a group of nine people, is it possible for each to be friends with exactly five others?
5. What is the minimum number of guests you need at a party to guarantee that either two people know each other or two people don't know each other?  
What about the same condition for three people? four people? five people?