

## Work on these with your partner(s) at the board

1. Define a sequence by  $a_1 = 1, a_2 = 3,$  and  $a_n = 3a_{n-1} - 2a_{n-2} \forall n \in \mathbb{N}, n \geq 3$ 
  - (a) Compute the first six terms of the sequence, i.e,  $a_1, \dots, a_6$
  - (b) Form a conjecture for the value of  $a_n$  that depends only on  $n$
  - (c) Use strong induction to prove your conjecture

*Ernst, Exercise 4.27*

2. Define the Fibonacci sequence by  $f_0 = 0, f_1 = 1,$  and  $f_n = f_{n-1} + f_{n-2} \forall n \in \mathbb{N}, n \geq 2$ 
  - (a) Compute the first six terms of the sequence, i.e,  $f_0, \dots, f_5$
  - (b) Use strong induction to prove that  $\left(\frac{3}{2}\right)^{n-2} \leq f_n \leq 2^n \forall n \in \mathbb{N}$
  - (c) Let  $\varphi = \frac{1 + \sqrt{5}}{2}$  and  $\psi = \frac{1 - \sqrt{5}}{2}.$  Prove that  $f_n = \frac{\varphi^n - \psi^n}{\sqrt{5}} \forall n \in \mathbb{Z}, n \geq 0$

*Based on Ernst, Exercise 4.29*