Prove each of the following. Explicitly list the type of proof used.

1. $\forall n \in \mathbb{Z}, n^2 \geq n$

2. Given any two consecutive integers, one is even and one is odd.

3. Given three consecutive integers, one is divisible by 3.

   Sketch: The integers will be of form $n, n+1, n+2$.
   Apply quotient/remainder property to n with $d = 3$, then use cases on r

4. Given four consecutive integers, one is divisible by 4.

5. If $p > 3$ is prime, then $p^2 - 1$ is divisible by 24.

   Hint: $p^2 - 1 = (p-1)(p+1)$, and apply previous two results

6. Prove that $\sqrt{2}$ is irrational.

Sketch:

- Suppose $\sqrt{2} \in \mathbb{Q}$, then $\sqrt{2} = \frac{a}{b}$ where $a, b \in \mathbb{Z},\ b \neq 0$.
- Argue that you can assume a and b are not both even.
- Do some more stuff. (This is like a Bake-Off technical challenge)
- Get a contradiction.
- Conclude $\sqrt{2}$ is irrational.