Work on these with your partner(s) at the board

1. Prove that the third Ramsey number $R(3) \neq 5$ by two-coloring the edges of K_5 such that there is no blue K_3 or red K_3 that is a subgraph.

2. Prove that $R(3) = 6$ by showing that every two-coloring of the edges of K_6 has a K_3 subgraph that is red or blue.

3. The purpose of this problem is to show that $R(4) > 17$.
   Label the vertices of K_{17} by $0, 1, 2, \ldots, 16$
   Color an edge connecting vertex i to vertex j red iff

$$(i - j) \mod 17 \equiv 1, 2, 4, 8, 9, 13, 15, \text{ or } 16$$

   Otherwise, color the edge blue.
   (a) Prove that there is no K_4 subgraph with red edges that contains vertex 0.
   (b) Prove that there is no K_4 subgraph with blue edges that contains vertex 0.
   (c) Argue by symmetry that the same argument works for all other vertices.

   Conclude that $R(4) > 17$.