Some Big Ideas, Week 8

Mar 18 – Mar 22, 2024

⊙ Review the summary of *Function Definitions* given on page 50 of Levin, Discrete Mathematics, An Open Introduction, 3rd edition.

⊙ A few notes about a function $f : X \longrightarrow Y$:

- The domain X of f is a *set*.
- The codomain Y of f is a *set*.
- The range of f is a *subset* of Y.
- If $x \in X$, then $f(x)$, the image of x, is a *single element* in Y.
- If $A \subseteq X$, then $f(A)$, the image of A, is a *subset* of Y.
- If $y \in Y$, then $f^{-1}(y)$, the preimage or inverse image of y, is a *subset* of X.
- If $B \subseteq Y$, then $f^{-1}(B)$, the preimage or inverse image of B, is a *subset* of X.

⊙ ***General structure to prove a function $f : X \longrightarrow Y$ is one-one (or injective):***

- Suppose that $x_1, x_2 \in X$ such that $f(x_1) = f(x_2)$.
- Show that $x_1 = x_2$.

⊙ ***General structure to prove a function $f : X \longrightarrow Y$ is onto (or surjective):***

- Let $y \in Y$ be an arbitrarily chosen element of Y.
- Show that $\exists\, x \in X$ such that $f(x) = y$.

⊙ **Definition:** If $f : X \longrightarrow Y$ is one-one and onto, then define the ***inverse function*** $f^{-1} : Y \longrightarrow X$ by $f^{-1}(y) = x$ iff $f(x) = y$.

⊙ **Definition:** If $f : X \longrightarrow Y$ and $g : Y' \longrightarrow Z$ where the range of f is a subset of Y', then define the ***composition*** $g \circ f : X \longrightarrow Z$ by $(g \circ f)(x) = g(f(x))$.

⊙ **Definition:** Sets A and B have the same ***cardinality*** iff there exists a bijection $f : A \longrightarrow B$.

Note: Compare this to the definition of cardinality given on page 50 of Levin.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the ***Tentative Weekly Syllabus*** on the course webpage for the specific sections relevant for this week.