Some Big Ideas, Week 5

Feb 19 – Feb 23, 2024

⊙ ***Principle of Mathematical Induction:***

Let $P(n)$ be a property that is defined for integers n. Let a be a fixed integer.

Suppose the following two statements are true:

1. $P(a)$ is true.
2. $\forall k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.

Then $\forall n \geq a, P(n)$ is true.

⊙ ***General structure of a Proof by Induction:***

Start by giving the statement that you want to prove:

Let $P(n)$ be the statement ...

To prove $P(n)$ is true for all $n \geq a$, requires two steps:

1. ***Base case:*** Prove that $P(a)$ is true.
2. ***Inductive case:*** Assume that $P(k)$ is true, and prove that $P(k+1)$ is true.
   "$P(k)$ is true" is called the ***inductive hypothesis***.

If you successfully proved both results, then you can conclude

Thus, by the principle of mathematical induction, $P(n)$ is true $\forall n \geq a$.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the ***Tentative Weekly Syllabus*** on the course webpage for the specific sections relevant for this week.