

Some Big Ideas, Week 4

Feb 13 – Feb 17, 2024

- ⊙ Some facts about numbers that we'll assume are true:
 - The basic properties of the algebra of real numbers hold, including the commutative, associative, and distributive laws, etc.
cf. Sundstrom, Table 1.2 pg 18; Ernst, Axioms 5.1, pg 61; Epp, Appendix A
 - The set of integers is closed under addition, subtraction, and multiplication.
Note that the integers are *not* closed under division since the quotient of two integers need not be an integer.
 - **Definition:** A number n is **even** if $\exists k \in \mathbb{Z}$ such that $n = 2k$.
 - **Definition:** A number m is **odd** if $\exists j \in \mathbb{Z}$ such that $m = 2j + 1$.
 - **Definition:** An integer $p > 1$ is **prime** iff the only natural numbers that evenly divide p are 1 and itself. A number that is not prime is called **composite**.
 - Given any $n \in \mathbb{Z}$ and any $d \in \mathbb{N}$, we can write $n = qd + r$ where $q, r \in \mathbb{Z}$ with $0 \leq r < d$.

- ⊙ A direct proof for a statement “ $\forall x \in D$, if $P(x)$, then $Q(x)$ ” will usually have the form:
 - Let $x \in D$ be an arbitrary value in the domain where $P(x)$ is true.
 - Argument, argument, argument.*
 - Conclude that $Q(x)$ is true.

- ⊙ To disprove a statement “ $\forall x \in D$, if $P(x)$, then $Q(x)$ ” by counterexample, find a specific $x \in D$ such that $P(x)$ is true and $Q(x)$ is false.

- ⊙ To prove a statement by contradiction:
 - Assume the negation of the statement is true.
 - Show that this logically leads to a contradiction.
 - Conclude that the statement is true.

- ⊙ To prove a statement “ $P(x) \rightarrow Q(x)$ ” by contraposition:
 - Assume $\sim Q(x)$ is true.
 - Argument, argument, argument.*
 - Conclude that $\sim P(x)$ is true.
 - Thus $\sim Q(x) \rightarrow \sim P(x)$, the contrapositive of the statement to be proved.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachele DeCoste at Wheaton.

Check the **Tentative Weekly Syllabus** on the course webpage for the specific sections relevant for this week.