

Some Big Ideas, Week 2

Jan 29 – Feb 2, 2024

- ⊙ **Definition:** A *proposition* or *statement* is a sentence that is true or false, but not both.

- ⊙ **Notation:** If p and q are propositions, then
 - $\sim p$ or $\neg p$ means “not p ”, also called the *negation* of p .
 - $p \vee q$ means “ p or q ”, also called the *disjunction* of p and q .
 - $p \wedge q$ means “ p and q ”, also called the *conjunction* of p and q .
 - $p \rightarrow q$ means “ p implies q ”, also called the *conditional*. p is the *hypothesis* and q is the *conclusion*

- ⊙ **Definition:** The *truth table* for a logical expression displays the truth values for the expression corresponding to all possible combinations of truth values for the propositions in the expression.

- ⊙ **Notation:** If expressions P and Q are logically equivalent, then we denote this with $P \equiv Q$.

- ⊙ **DeMorgan’s Laws:** If P and Q are logical expressions, then
 - $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
 - $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

- ⊙ **Definition:** The *biconditional* “ p if-and-only-if q ” means $(p \rightarrow q) \wedge (q \rightarrow p)$. We will often abbreviate to “ p iff q ”.

- ⊙ **Definition:** The *contrapositive* of a conditional statement “If p then q ” is the statement “If $\neg q$ then $\neg p$ ”.

- ⊙ **Definition:** The *converse* of a conditional statement “If p then q ” is the statement “If q then p ”.

- ⊙ **Definition:** The *inverse* of a conditional statement “If p then q ” is the statement “If $\neg p$ then $\neg q$ ”.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the *Tentative Weekly Syllabus* on the course webpage for the specific sections relevant for this week.