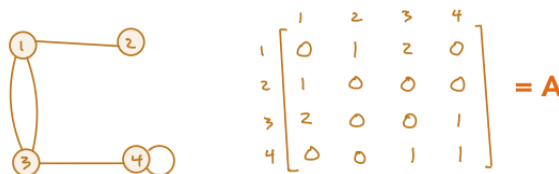


Some Big Ideas, Week 13

Apr 22 – Apr 26, 2024

- ⊙ **Definition:** An *isomorphism* between graphs G_1 and G_2 is a bijection $f : V_1 \rightarrow V_2$ between the vertices of the graphs such that $\{a, b\}$ is an edge in G_1 iff $\{f(a), f(b)\}$ is an edge in G_2 .
- ⊙ **Definition:** A property P is called an *invariant for graph isomorphisms* iff given any graphs G and G' , if G has property P and G' is isomorphic to G , then G' also has property P .
- ⊙ **Definition:** For any graph G , we can define the *adjacency matrix* $A = (a_{ij})$ where

$$a_{ij} = \# \text{ edges from vertex } i \text{ to vertex } j$$



Note that If G is undirected, then the matrix A will be symmetric. i.e. $a_{ij} = a_{ji}$

- ⊙ **Definition:** A *coloring* of a graph is an assignment of labels, called *colors*, to the vertices of the graph.
A *proper coloring* is a coloring such that adjacent vertices are never assigned the same color.
- ⊙ **Definition:** The *chromatic number* of a graph G , denoted $\chi(G)$, is the minimum number of colors needed to properly color a graph.
- ⊙ **Definition:** A *clique* of a graph G is a subgraph of G that is a complete graph.
That is, for any two vertices in the subgraph, there is an edge in the subgraph connecting them.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachele DeCoste at Wheaton.

Check the *Tentative Weekly Syllabus* on the course webpage for the specific sections relevant for this week.