Some Big Ideas, Week 11

Apr 8 – Apr 12, 2024

- ***The Pigeonhole Principle:*** If A and B are finite sets where $|A| > |B|$, then there is no one-one function from A to B.

  i.e. There must exist two elements of A that map to the same value in B.

- ***Generalize Pigeonhole Principle:*** If A and B are finite sets where $|A| = n$ and $|B| = m$, then for any positive integer $k < \frac{n}{m}$, there exists some $b \in B$ such that b is the image of at least $k + 1$ distinct elements of A.

- **Definition**: A ***k-combination*** of a set A is a subset of A consisting of k elements.

  If A has n elements, then the number of k-combinations of A is denoted by $\binom{n}{k}$.

- **Theorem**: For all non-negative integers with $k \leq n$,

$$\binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{k!(n-k)!}$$

- **Pascal's Theorem**: For all positive integers with $k \leq n$,

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

- **The Binomial Theorem**: For all $a, b \in \mathbb{R}$ and any non-negative integer n,

$$\begin{aligned}(a+b)^n &= \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \\ &= a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n-1} a^1 b^{n-1} + b^n\end{aligned}$$

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the ***Tentative Weekly Syllabus*** on the course webpage for the specific sections relevant for this week.