

Some Big Ideas, Week 10

Apr 1 – Apr 5, 2024

- ⊙ **Definition:** A *sample space* is the set of all possible outcomes of an experiment or random process. An *event* is a subset of a sample space.

- ⊙ **Definition:** If S is a finite sample space where all outcomes are equally likely and E is an event in S , then the *probability of E* is

$$P(E) = \frac{\text{number of outcomes in } E}{\text{total number of outcomes in } S} = \frac{|E|}{|S|}$$

Note: Epp uses the notation $N(A)$ for the cardinality of a finite set rather than $|A|$.

- ⊙ **Additive Principle:** If an event E can occur in m ways and the event F can occur in n ways where E and F are disjoint, then the event “ E or F ” can occur in $m + n$ ways.

- ⊙ **Multiplicative Principle:** If an event E can occur in m ways, and each possibility for E allows for exactly n ways for the event F to occur, then the event “ E and F ” can occur in $m \cdot n$ ways.

- ⊙ **Definition:** A *permutation* of a set A is an ordered, non-repetitive arrangement of all elements of A .

For example, if $A = \{a, b, c\}$, then there are six permutations of A : $abc, acb, cab, cba, bca, bac$

- ⊙ **Definition:** A *k -permutation* of A an ordered, non-repetitive arrangement of k elements of A .

For example, if $A = \{a, b, c, d\}$, a 2-permutation of A is bc and a 3-permutation is cad .

- ⊙ **Theorem (Epp 9.2.3):** The number of k -permutations from a set with n elements, denoted $P(n, k)$, is

$$P(n, k) = \frac{n!}{(n - k)!}$$

- ⊙ If A and B are finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachele DeCoste at Wheaton.

Check the *Tentative Weekly Syllabus* on the course webpage for the specific sections relevant for this week.