Problem Set #8

Due Friday, April 12, 2024 @ 12:30 pm
Submit as single pdf file to Canvas

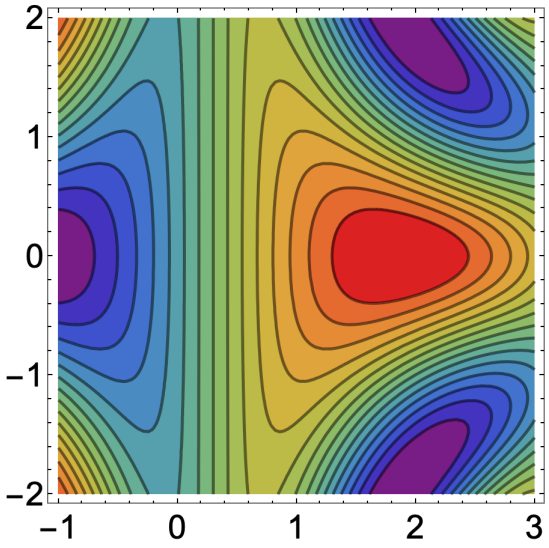
Remember to review the Guidelines for WeBWorK and Problem Sets on the course webpage when writing up your solutions. The rule of thumb is that you should give enough explanation so that you could hand your assignment to a student who took Calc II last semester and they could follow your solutions.

1. Let $r(t) = \langle \cos(t), \sin(4t) \rangle$
   (a) Find $r'(t)$
   (b) Sketch the graph of $r(t)$
   (c) On your sketch, label the point $r(0)$ and sketch the vector $r'(0)$
   Use $r(0)$ as the base point for $r'(0)$
   (d) Repeat (c), labeling $r(t)$ and $r'(t)$ for $t = \frac{\pi}{8}, \frac{\pi}{4}, \pi,$ and $\frac{5\pi}{2}$

   Note that this problem is very similar to the In-Class work from April 5

2. Use the contour plot of $g(x, y)$ to answer the following.

   Contour plot of $g(x, y)$



   Blue represents lower values and
   Red represents higher values

   (a) Will the directional derivative of $g(x, y)$ at the point $(x, y) = (0, 1)$ in the direction of $v = \langle -1, -1 \rangle$ be positive, negative or zero? Explain.
   (b) If you are standing at the point on the surface with $(x, y) = (0, 0)$, in what direction should you walk to go uphill the fastest? Why?
   (c) If you are standing at the point on the surface with $(x, y) = (0, 0)$, in which direction does the gradient point? Why?
   (d) If you are standing at the point on the surface with $(x, y) = (1, -1)$, in which direction does the gradient point? Why?

3. Let $f(x, y) = xe^y + x^2y - 2y$.
   (a) Find ∇f, the gradient of $f(x, y)$
   (b) If $\vec{u} = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$, compute $D_u f(-1, 4)$, the directional derivative of $f(x, y)$ in the direction of $\vec{u}$
   (c) Find the directional derivative of $f(x, y)$ at $(-1, 4)$ in the direction of $\vec{v} = \langle 2, 5 \rangle$.
   Note that $\vec{v}$ is not a unit vector!

4. Let $h(x, y) = x^2 + 2y^2 - xy - 7x + 19$
   (a) At the point $(2, 1)$, what is the direction in which $h(x, y)$ is increasing the fastest?
   (b) What is rate of change of $h(x, y)$ at the point $(2, 1)$ in the direction of $\vec{v} = \langle 2, -3 \rangle$?
   (c) If you are standing on the surface at the point $(x, y, z) = (3, 2, 9)$ and drop a ball, in which direction will it roll? Why?