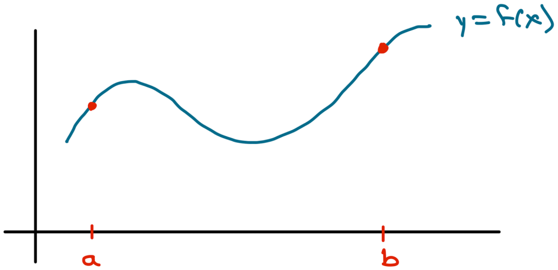
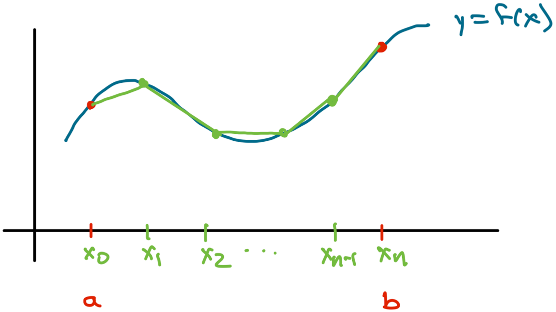
Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$



Since $f(x)$ is differentiable, locally it is locally linear

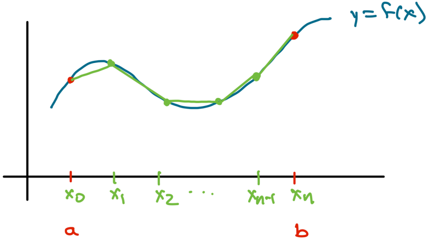
Thus, it makes sense to approximate the arc length by straight lines

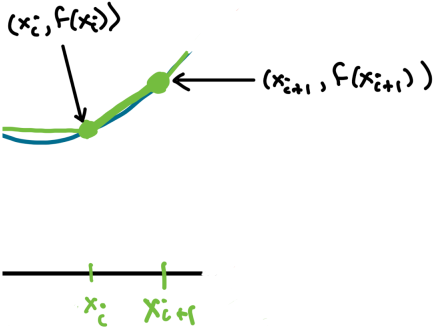
Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$



Subdivide $[a, b]$ into n subintervals of width Δx with $a = x_0 < x_1 < \ldots < x_n = b$

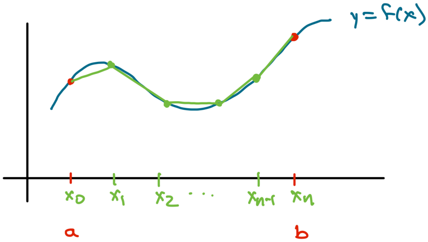
Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

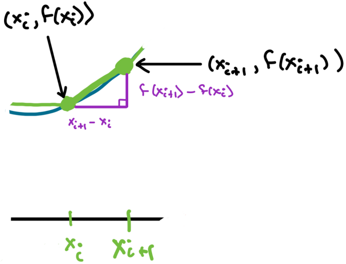




What is the length of the segment that joins $(x_i, f(x_i))$ and $(x_{i+1}, f(x_{i+1}))$?

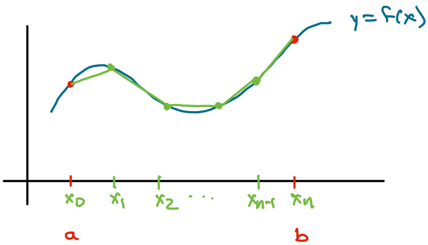
Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

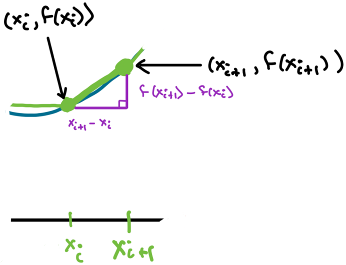




$$\text{Length of segment} = \sqrt{(x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2}$$

Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

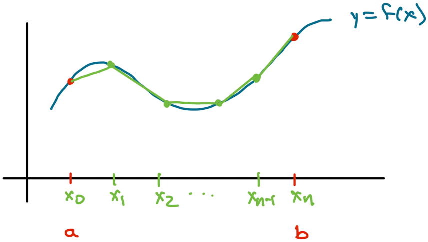


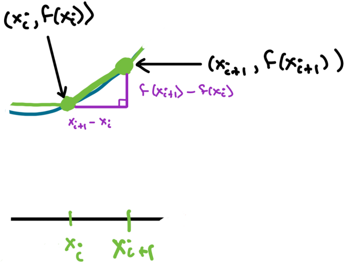


$$\text{Length of segment} = \sqrt{(x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2}$$

$$= \sqrt{\left((x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2\right) \frac{(x_{i+1} - x_i)^2}{(x_{i+1} - x_i)^2}}$$

Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$





$$\text{Length of segment} = \sqrt{\left((x_{i+1} - x_i)^2 + (f(x_{i+1}) - f(x_i))^2\right) \frac{(x_{i+1} - x_i)^2}{(x_{i+1} - x_i)^2}}$$

$$= \sqrt{1 + \left(\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}\right)^2} \quad (x_{i+1} - x_i)$$

Mean Value Theorem

If f is differentiable on $[a, b]$, then there is at least one $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

i.e. If your avg velocity on a trip is 60 mph, at some point you were going *exactly* 60 mph.

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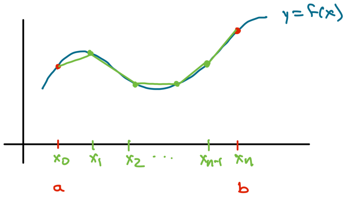
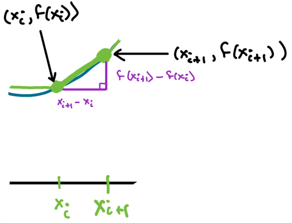
Applying the MVT to the interval $[x_i, x_{i+1}]$, there is a $c_i \in [x_i, x_{i+1}]$ such that

$$f'(c_i) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$



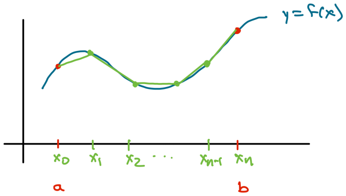


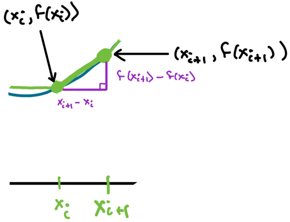
$$\text{Length of segment} = \sqrt{1 + \left(\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}\right)^2}\ (x_{i+1} - x_i)$$

$$= \sqrt{1 + (f'(c_i))^2}\ \Delta x$$

$$\Rightarrow \qquad \text{Arc Length} \approx \sum_{i=1}^{n} \sqrt{1 + (f'(c_i))^2}\ \Delta x$$

Why $AL = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$





$$\text{Arc Length} \approx \sum_{i=1}^{n} \sqrt{1 + (f'(c_i))^2}\ \ \Delta x$$

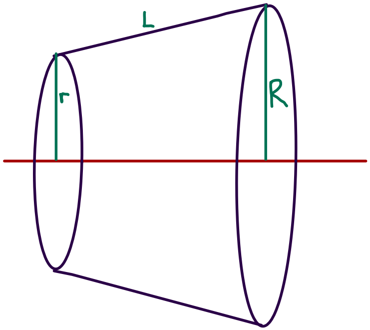
$$\Rightarrow \qquad \text{Arc Length} = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + (f'(c_i))^2}\ \ \Delta x$$

$$= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Why $SA = \int_a^b 2\pi\, f(x)\, \sqrt{1 + (f'(x))^2}\, dx$

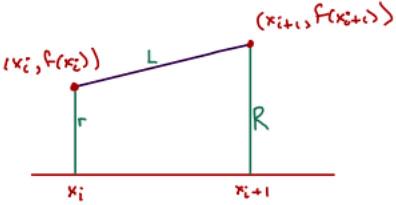
Play same game, approximate $y = f(x)$ with straight line segments.

Use the formula for the surface area of the frustum of a cone:



$SA = \pi(R+r)L$

Why $SA = \int_a^b 2\pi\, f(x)\, \sqrt{1+(f'(x))^2}\, dx$



Surface area of frustum

$= \pi(R+r)L$

$= \pi\left(f(x_{i+1}) + f(x_i)\right)\sqrt{(x_{i+1}-x_i)^2 + (f(x_{i+1})-f(x_i))^2}$

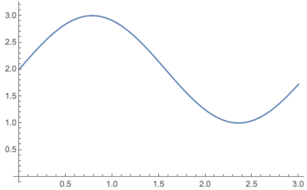
$= \pi\left(f(x_{i+1}) + f(x_i)\right)\sqrt{1+(f'(c_i))^2}\ \Delta x$

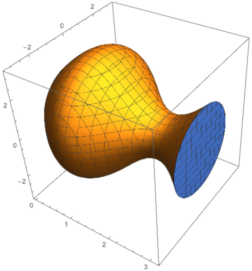
Thus,

$$SA \approx \sum_{i=1}^{n} \pi(f(x_{i+1}+f(x_i))\ \sqrt{1+(f'(c_i))^2}\ \ \Delta x$$

$$\Rightarrow \quad SA = \int_a^b 2\pi\, f(x)\, \sqrt{1+(f'(x))^2}\, dx$$

Let C be the graph of $y = \sin(2x) + 2$ for $0 \leq x \leq \pi$





1. Set up the integral that give the arc length of C and use Simpson's rule with 50 subdivisions to approximate the arc length. How accurate is your approximation?

2. Set up the integral that give the surface area of the solid formed when C is rotated about the x-axis. Approximate the surface area accurate within 0.001 of its exact value using Simpsons rule