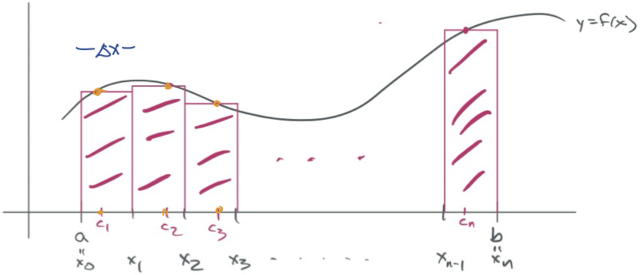
Recall: We use Riemann Sums to define an integral $\int_a^b f(x)\ dx$



$$\int_a^b f(x)\,dx$$

$$\text{Area} \approx f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x$$

$$= \sum_{k=1}^{n} f(c_k)\Delta x$$

Get more accurate by taking more subdivisions!

i.e.

$$\text{Area} = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x$$

We (usually) use the Fundamental Theorem of Calculus to evaluate an integral

Let f be continuous on $[a, b]$ and let F be *any* antiderivative of f. Then

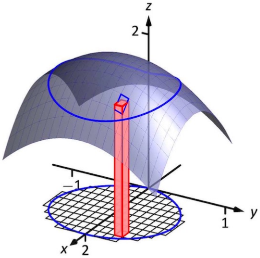
$$\int_a^b f(x)\, dx = F(b) - F(a)$$

If cannot find an elementary antiderivative $F(x)$, then can use

- numeric techniques, like Simpson's Rule, or
- possibly a Taylor series for $f(x)$

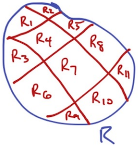
Similarly: We use Riemann Sums to define an integral $\iint_R f(x,y)\, dA$

$\iint_R f(x,y)\ dA$ represents the signed volume between $z = f(x,y)$ and the xy-plane over the region R



(b)

Figure 13.2.1: Developing a method for finding signed volume under a surface.



$$\iint_R f(x,y)\ dA = \text{Volume}$$

$$\approx f(u_1,v_1)\Delta A_1 + \cdots f(u_n,v_n)\Delta A_n$$

$$= \sum_{k=1}^{n} f(u_k,v_k)\Delta A_k$$

$$\Rightarrow \iint_R f(x,y)\ dA = \lim_{n\to\infty} \sum_{k=1}^{n} f(u_k,v_k)\Delta A_k$$

Can use earlier intuition about finding volumes via cross-sections

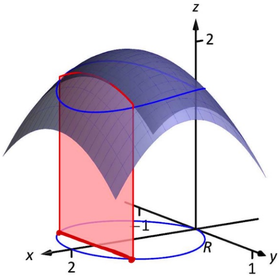
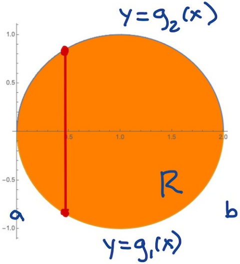
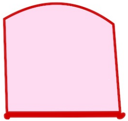


Figure 13.2.2: Finding volume under a surface by sweeping out a cross–sectional area.



$$A(x) = \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy$$



$$\iint_R f(x, y)\, dA = \text{Volume} \quad = \int_a^b A(x)\, dx$$

$$= \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

Talk with the people around you for a minute

If $\iint_R f(x,y)\ dA = \int_1^5 \int_{-1}^1 f(x,y)\ dx\ dy$ then R is

(a) a square

(b) a rectangle

(c) a triangle

(d) a circle

(e) something else

Talk with the people around you for a minute

If $\iint_R f(x,y)\ dA = \int_0^3 \int_{-x+5}^5 f(x,y)\ dy\ dx$ then R is

(a) a square

(b) a rectangle

(c) a triangle

(d) a circle

(e) something else

Talk with the people around you for a minute

If $\iint_R f(x,y)\ dA = \int_{-3}^{2} \int_{y^2}^{-y+6} f(x,y)\ dx\ dy$ then R is

(a) a square

(b) a rectangle

(c) a triangle

(d) a circle

(e) something else

1. Find the volume below the surface $z = 1 + x + y$ and above the region R in the xy-plane bounded by the graphs $x = 1, y = 0, y = x^2$.

2. Find the volume below the surface $z = \cos(x^2)$ and above the triangle R in the xy-plane bounded by the x-axis, the line $x = 1$, and the line $y = x$.

3. Evaluate $\iint_R 2x - y \; dA$ where R is the region in the xy-plane bounded by the x-axis and the upper half of the circle centered at the origin with radius 2.

4. Sketch the region determined by the bounds and reverse the order of integration

(a) $\displaystyle\int_0^1 \int_y^{\sqrt[3]{y}} f(x, y)\ dx\ dy$

(b) $\displaystyle\int_{-\sqrt{2}}^0 \int_0^{-x^2+2} f(x, y)\ dy\ dx \ + \ \int_0^4 \int_0^{-\frac{1}{2}x+2} f(x, y)\ dy\ dx$

(You can combine this to one integral!)

(c) $\displaystyle\int_0^3 \int_{x-3}^{-x+3} f(x, y)\ dy\ dx$