

1. Let $f(x, y) = xy$

1. Find $\nabla f(x, y)$
2. Find the directional derivative of f at the point $P = (-2, 1)$ in the direction of the given vector \vec{v} :

(a) $\vec{v} = \langle 1, 0 \rangle$

(d) $\vec{v} = \langle 1, -2 \rangle$

(b) $\vec{v} = \langle 0, 1 \rangle$

(e) $\vec{v} = \langle 2, 1 \rangle$

(c) $\vec{v} = \langle -1, 1 \rangle$

(f) $\vec{v} = \langle -1, 2 \rangle$

3. Verify your results by looking at a contour plot of $f(x, y)$

2. Let $f(x, y) = 5 - 10xy - 4x^2 + 3y - y^4$

Suppose you are standing at the point $(2, 1)$ on the surface $z = f(x, y)$

1. If you walk in the given direction, will you be going uphill or downhill?
At what rate?
(a) $\vec{u} = \langle 1, 0 \rangle$ (b) $\vec{u} = \langle 0, 1 \rangle$ (c) $\vec{v} = \langle -1, -2 \rangle$ (d) $\vec{w} = \langle 2, -3 \rangle$
2. In which direction should you walk to move uphill at the fastest rate?
What is the rate?
3. If you spill your milk, in which direction will it flow? What is the rate?
4. What direction should you walk to stay at the same altitude?
Is there more than one direction?
5. Make a contour plot to verify that your answers are plausible
Note: You may need to adjust the range values and the number of contours