Problem Set #7

Due Friday, November 8, 2024 @ 12:30 pm
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course webpage when writing up your solutions!*

1. Let $A = \begin{bmatrix} 9 & -4 & -2 & -4 \\ -56 & 32 & -28 & 44 \\ -14 & -14 & 6 & -14 \\ 42 & -33 & 21 & -45 \end{bmatrix}$

   (a) Find the eigenvalues of A.

   (b) Find a basis for the eigenspace of each eigenvalue you found in part (a).

2. Let $A = \begin{bmatrix} -2 & 3 & -12 & 0 & 4 \\ -2 & 1 & -8 & 2 & 4 \\ 1 & 1 & 1 & 2 & 7 \\ 1 & -1 & 5 & 0 & -1 \end{bmatrix}$

   (a) Find a basis for the orthogonal complement of $\text{row}(A)$.

   (b) Find a basis for the orthogonal complement of $\text{col}(A)$.

3. Let $\vec{\mathbf{u_1}} = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{\mathbf{u_2}} = \begin{bmatrix} 2 \\ 2 \\ -1 \\ 1 \end{bmatrix}$, and $\vec{\mathbf{y}} = \begin{bmatrix} -7 \\ -16 \\ 8 \\ 4 \end{bmatrix}$

   (a) Verify that $\vec{\mathbf{u_1}}$ and $\vec{\mathbf{u_2}}$ are orthogonal.

   (b) Find the orthogonal projection of $\vec{\mathbf{y}}$ onto $\vec{\mathbf{u_1}}$.

   (c) Find the orthogonal projection of $\vec{\mathbf{y}}$ onto $\vec{\mathbf{u_2}}$.

   (d) Verify $\vec{\mathbf{y}} \in \text{Span}\{\vec{\mathbf{u_1}}, \vec{\mathbf{u_2}}\}$ and write $\vec{\mathbf{y}}$ as a linear combination of $\vec{\mathbf{u_1}}$ and $\vec{\mathbf{u_2}}$.

   (e) How is your answer to (d) related to your answers to (b) and (c)?