Problem Set #6

Due Friday, November 1, 2024 @ 12:30 pm
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course webpage when writing up your solutions!*

1. Suppose A is the matrix corresponding to an onto linear transformation $T : \mathbb{R}^7 \to \mathbb{R}^3$.
   (a) What is the dimension of $\text{nul}(A)$? $\text{col}(A)$? Why?
   (b) What is $\text{range}(T)$? Why?
   (c) Describe $\text{col}(A^T)$ geometrically.

2. Let $P = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0.2 & 0 & 0.8 \\ 0.5 & 0.9 & 0.2 \end{bmatrix}$
   (a) Show that P is a regular stochastic matrix.
   (b) Find the steady-state vector for P.

3. Let $A = \begin{bmatrix} -18 & -63 \\ 6 & 21 \end{bmatrix}$
   (a) Verify that $x = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ is an eigenvector of A with corresponding eigenvalue $\lambda = 3$
   (b) Is $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ an eigenvector of A? Why or why not?
   (c) What is the characteristic equation of A?
   (d) Find all the eigenvalues of A.
   (e) For each eigenvalue you found in part (d), other than $\lambda = 3$, find a corresponding eigenvector.