

## PROBLEM SET #4

Due Friday, October 11, 2024 @ 12:30 pm  
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course webpage when writing up your solutions!*

1. Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the transformation that rotates the plane by  $\frac{\pi}{3}$  radians clockwise about the point  $(3, -4)$ .
  - (a) Explain why  $T$  is not a *linear* transformation.
  - (b) Find the  $3 \times 3$  matrix  $A$  that produces  $T$  using homogeneous coordinates.
  - (c) What is the image of the point  $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$  under  $T$ ?

$$2. \text{ Let } A = \begin{bmatrix} 0 & 3 & 2 & 1 \\ 1 & 3 & 0 & 2 \\ 5 & 4 & 0 & 3 \\ 3 & 0 & 1 & 2 \end{bmatrix}$$

- (a) Compute  $\det(A)$  by hand using a cofactor expansion.  
You may pick which row or column you expand along.
- (b) Show that  $A$  is invertible.
- (c) What is  $\det(A^{-1})$ ? Note that you don't need to actually find  $A^{-1}$ !

$$3. \text{ Let } A = \begin{bmatrix} -5 & 3 & 9 & 8 & 2 & 11 & -2 & 10 \\ -11 & 7 & 0 & 8 & 0 & -8 & 6 & -9 \\ -2 & 12 & 0 & 7 & 9 & 12 & 1 & -5 \\ -2 & 3 & 3 & 6 & 4 & 4 & -3 & -6 \\ 9 & 5 & -1 & -11 & -9 & 2 & -6 & 10 \\ 9 & 5 & -6 & 0 & -11 & 7 & -8 & -11 \\ 0 & -2 & 12 & -3 & 2 & 7 & 1 & -9 \\ -5 & -8 & -5 & 4 & 10 & -3 & -11 & 9 \end{bmatrix}$$

Notice that  $A$  is given in Mathematica format on the Problem Sets page so that you can copy and paste into your Mathematica notebook.

- (a) What is the determinant of  $A$ ? Please don't do this by hand!
  - (b) Is  $A$  invertible? Why or why not?
  - (c) Is the linear transformation  $T : \mathbb{R}^8 \rightarrow \mathbb{R}^8$  defined by  $T(\vec{x}) = A\vec{x}$  one-one? Explain.
4. Let  $H = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \mid a, b \leq 0 \right\}$ , the 3<sup>rd</sup> quadrant in  $\mathbb{R}^2$ .
    - (a) Is  $H$  closed under vector addition? Explain.
    - (b) Is  $H$  closed under scalar multiplication? Explain.
    - (c) Is  $H$  a subspace of  $\mathbb{R}^2$ ? Explain.

$$5. \text{ Let } B = \begin{bmatrix} 3 & 6 & 15 \\ 6 & 12 & 29 \\ 3 & 6 & 13 \end{bmatrix}$$

- (a) Give a non-zero vector in  $\text{nul}(B)$ . Explain.
- (b) Give a non-zero vector in  $\text{col}(B)$ . Explain.

- (c) Is  $\vec{\mathbf{b}} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix}$  in  $\text{col}(B)$ ? Explain.
- (d) Give a vector in  $\mathbb{R}^3$  that is **not** in  $\text{col}(B)$ . Explain.