Problem Set #2

Due Friday, September 13, 2024 @ 12:30 pm
Submit as single pdf file to Canvas

*Remember to review the **Guidelines for Problem Sets** on the course webpage when writing up your solutions!* You may use Mathematica for your calculations unless you are explicitly told to do the computation by hand. When you use Mathematica, be sure to take a screen shot of the relevant parts of your notebook to include in your writeup.

1. Consider the augmented matrix $\left[\begin{array}{ccc|c} 44 & 89 & 6 & -357 \\ -16 & -32 & -2 & 130 \\ 10 & 21 & 2 & -80 \end{array}\right]$

   (a) This augmented matrix corresponds to a system of linear equations in three variables. What is the system of equations?

   (b) This augmented matrix corresponds to a vector equations in three variables. What is the vector equation?

   (c) This augmented matrix corresponds to a matrix equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$. What are A and $\vec{\mathbf{b}}$?

   (d) Solve the system, and give your answer as a solution to the system from part (a).

2. Let $A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -6 & 1 & -7 \\ 3 & 9 & -4 & 18 \\ 1 & 3 & 1 & -1 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 7 \\ -23 \\ 57 \\ -2 \end{bmatrix}$.

   (a) Write the general solution to $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ in parametric form.

   (b) Are the columns of A linear independent or linearly dependent? Explain.

   (c) Do the columns of A span $\mathbb{R}^4$? Explain.

   (d) Does $\vec{\mathbf{b}}$ lie in the span of the columns of A? Explain.

3. Each statement is either true (in all cases) or false (for at least one example). If false, construct a specific counterexample to show that the statement is not always true. If a statement is true, give a justification. (One specific example cannot explain why a statement is always true.)

   (a) The columns of every 3×5 matrix A are linearly dependent.

   (b) If $\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}, \vec{\mathbf{v_3}}$ are in $\mathbb{R}^3$ and $\vec{\mathbf{v_3}}$ is *not* a linear combination of $\vec{\mathbf{v_1}}$ and $\vec{\mathbf{v_2}}$ then $\{\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}, \vec{\mathbf{v_3}}\}$ is linearly independent.

   (c) If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are linear independent and $\vec{\mathbf{w}}$ lies in Span $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$, then $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}\}$ is linearly dependent.

   (The problem is very similar to Exercises 1.7.33-38 from the text, Lay's Linear Algebra, 4th edition)