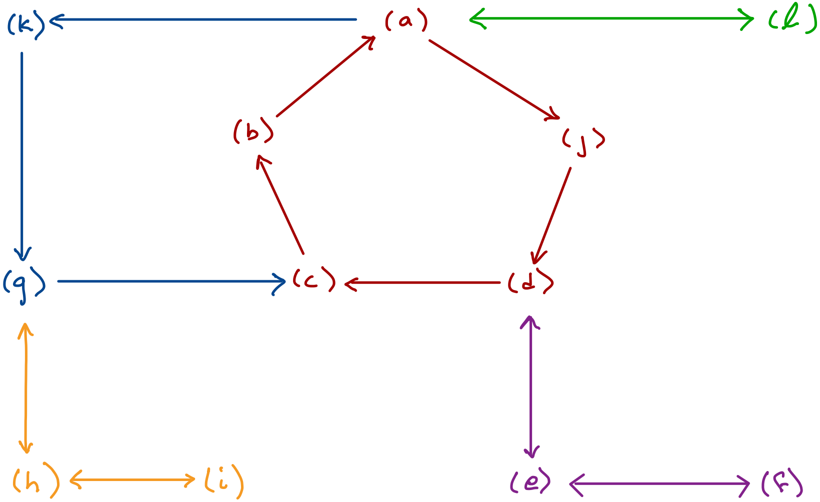
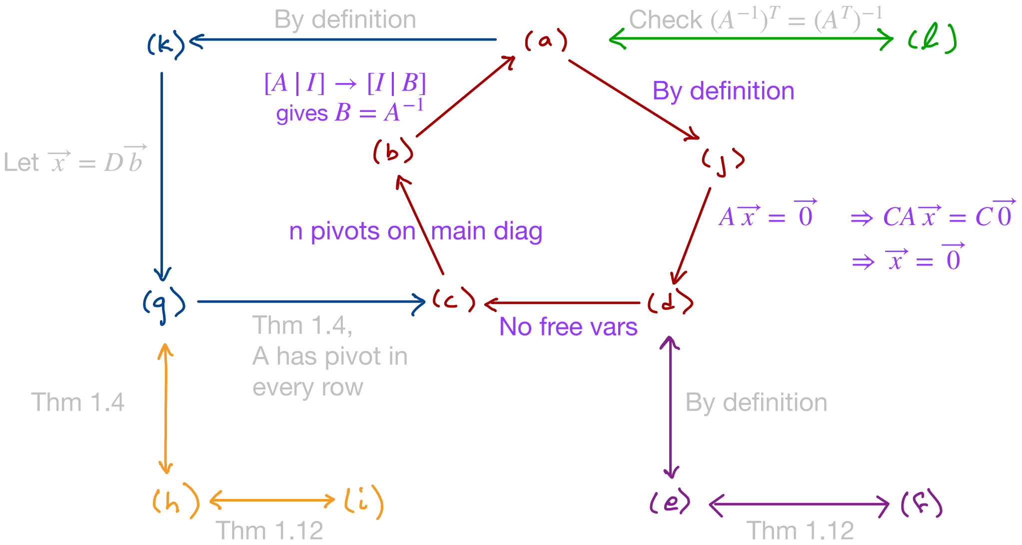
The Invertible Matrix Theorem (Thm 2.8): Let A be a square $n \times n$ matrix. Then the following statements are equivalent.

a. A is an invertible matrix.
b. A is row equivalent to the $n \times n$ identity matrix.
c. A has n pivot positions.
d. The equation $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ has only the trivial solution.
e. The columns of A form a linearly independent set.
f. The linear transformation $\vec{\mathbf{x}} \to A\vec{\mathbf{x}}$ is one-to-one.
g. The equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$. has at least one solution for each $\vec{\mathbf{b}}$ in $\mathbb{R}^n$.
h. The columns of A span $\mathbb{R}^n$.
i. The linear transformation $\vec{\mathbf{x}} \to A\vec{\mathbf{x}}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.
j. There is an $n \times n$ matrix C such that $CA = I$.
k. There is an $n \times n$ matrix D such that $AD = I$.
l. A^T is an invertible matrix.

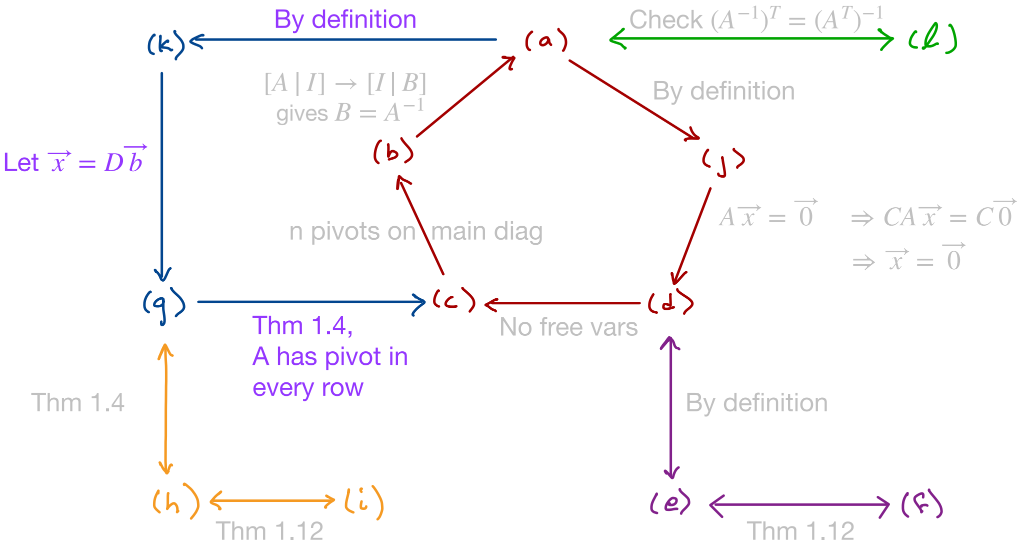
Sketch of proof



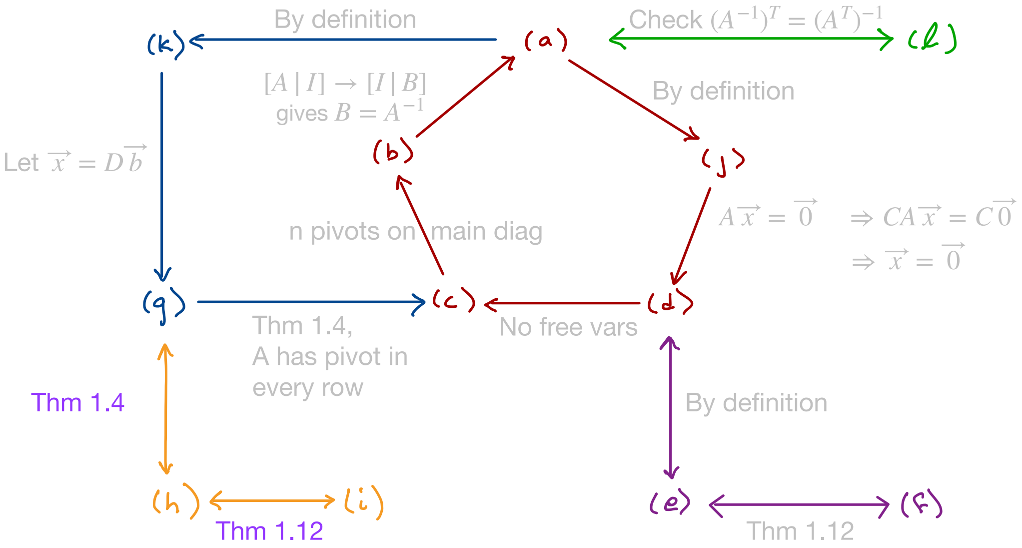
Sketch of proof



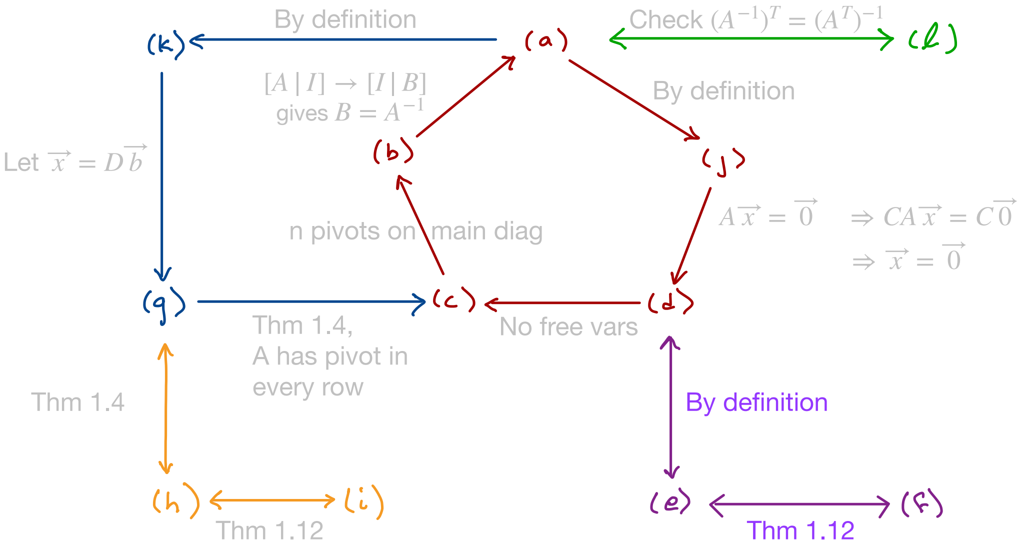
Sketch of proof



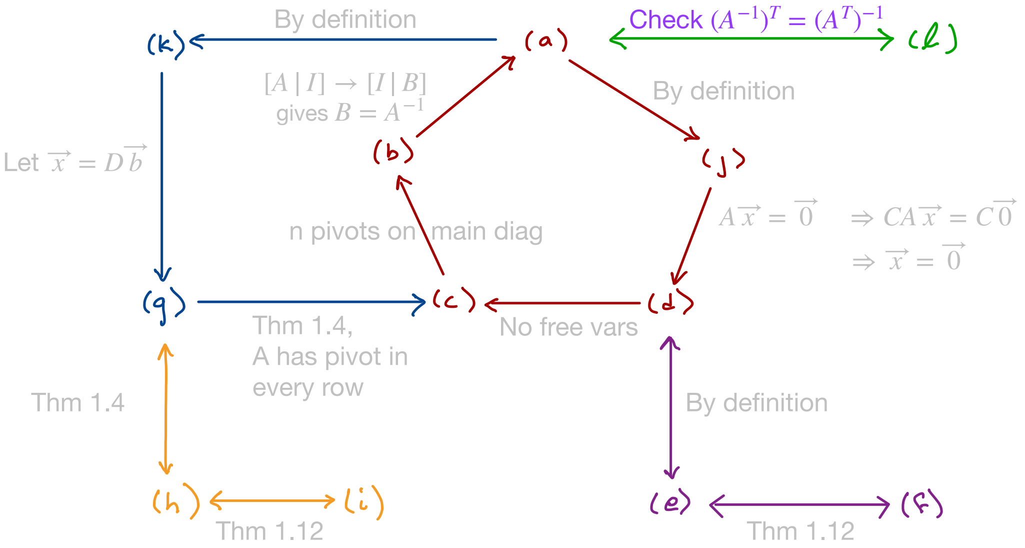
Sketch of proof



Sketch of proof



Sketch of proof



Sketch of proof